Simultaneous Inferences

- **Simultaneous Inference of β_0 and β_1.**

  Example: Consider the Toluca Company example. Obtain a 90% family confidence intervals for β_0 and β_1.

- **Bonferroni Procedure.** $\hat{\theta} \pm B * s\{\hat{\theta}\}$, where $B = t(1 - \alpha/(2 * g); df = n - 2)$

```
data=read.csv("Toluca.csv",header=T)
attach(data)

results=lm(hours~size)
confint(results,level=.90)
coefficients(summary(results))

b0=coefficients(summary(results))[1,1]  # Extracting the value of b0
se.b0=coefficients(summary(results))[1,2] # The standard error of b0

b1=coefficients(summary(results))[2,1] # Extracting the value of b1
se.b1=coefficients(summary(results))[2,2] # The standard error of b1

# The 90% joint confidence intervals for beta_0 and beta_1:
B=qt(1-.10/(2*2),df=23) # In general, B=qt(1-.10/(2*g),df), g=no. of intervals
lower.b0=b0-B*se.b0;upper.b0=b0+B*se.b0
lower.b1=b1-B*se.b1;upper.b1=b1+B*se.b1

# Or you can make the Bonferroni adjustment here
confint(results,level=(1-.10/2)) # In general, confint(results,level=(1-alpha/g))
```

- **Simultaneous Estimation of Mean Responses**.

  Example: Consider the Toluca Company example, suppose we require a family of estimates of the mean number of work hours at the following lot size levels: $X_h = 30, 65, 100$. The family confidence coefficient is to be 0.90.

```
new=data.frame(size=c(30,65,100))
ci=predict(results,new,interval="confidence",se.fit=T)

# Using Bonferroni Procedure
B=qt(1-.10/(2*3),23)

ci$fit[1]-B*ci$se[1]; ci$fit[1]+B*ci$se[1] # at size=30
ci$fit[2]-B*ci$se[2]; ci$fit[2]+B*ci$se[2] # at size=65
ci$fit[3]-B*ci$se[3]; ci$fit[3]+B*ci$se[3] # at size=10

# Or you can make the Bonferroni adjustment here
predict(results,new,interval="confidence",level=(1-.10/3))
```

- **Working-Hotelling Procedure.** $\hat{Y}_h \pm W * s\{\hat{Y}_h\}$, where $W^2 = 2 * F(1 - \alpha; df1 = 2, df2 = n - 2)$

```
# Using Working-Hotelling Procedure
# to constuct 90% joint confidence intervals for the mean response at the 3 x levels.
W2=2*qf(1-.10,2,23)
```

```
ci$fit[1]-sqrt(W2)*ci$se[1]; ci$fit[1]+sqrt(W2)*ci$se[1] # at size=30
ci$fit[2]-sqrt(W2)*ci$se[2]; ci$fit[2]+sqrt(W2)*ci$se[2] # at size=65
ci$fit[3]-sqrt(W2)*ci$se[3]; ci$fit[3]+sqrt(W2)*ci$se[3] # at size=100
```

- **Simultaneous Prediction Intervals for New Observations**.

  Example: The Toluca Company wishes to predict the work hours required for each of the next two lots, which will consist of 80 and 100 units. The family confidence coefficient is to be 95%.

```
# Bonferroni Procedure
results=lm(hours~size)
new=data.frame(size=c(80,100))

ci.bon=predict(results,new,interval="prediction",level=(1-.05/2))
```

- **Scheffe Procedure.** $\hat{Y}_h \pm S * s\{pred\}$, where $S^2 = g * F(1-\alpha; df1 = g, df2 = n-2)$

```
# Scheffe Procedure
mse=anova(results)$Mean[2]
ci=predict(results,new,interval="prediction",se.fit=T)

se.fit.pred=sqrt(ci$se.fit^2 + mse))

S=sqrt(2*qf(1-.05,2,23)) # In general, use S=sqrt(k*qf(1-.05,k,n-2))
# If k=2:10, look at plot(k,sqrt(k*qf(1-.05,k,n-2)))
lwr.scheffe=ci$fit[,1]-S*se.fit.pred
upr.scheffe=ci$fit[,1]+S*se.fit.pred
cbind(ci$fit,lwr.scheffe,upr.scheffe)
```