Two-Factor Studies

- **Examples of Crossed, Two-Factor Designs**

    1. A company investigated the effects of selling price and type of promotional campaign on sales volume of one of its products. Three selling prices (55 cents, 60 cents, 65 cents) were studied, as were two types of promotional campaigns (radio advertising, newspaper advertising). Twelve communities throughout the United States, of approximately equal size and similar socioeconomic characteristics, were selected and the treatments were assigned to them at random, such that each treatment was given to two communities.

        a. Is this an example of *experimental* or *observational* study? _____

        b. What study design was employed in this study? _____

        c. Identify the response variable. _____

        d. Identify the Experimental units. _____

        e. Identify the Factors and their levels. _____

        f. How many treatments are there? _____

    2. An analyst studied the effects of family income (under \$15,000, \$15,000 − \$29,999, \$30,000 − \$49,999, \$50,000 and more) and stage in the life cycle of the family (stages 1, 2, 3, 4) on amount spent on appliance purchases in the last 5 years. The analyst selected 20 families with the required income and life-cycle characteristics for each of the "treatment" classes for this study, yielding 320 families for the entire study.

        a. Is this an example of *experimental* or *observational* study? _____

        b. What study design was employed in this study? _____

        c. Identify the response variable. _____

        d. Identify the Experimental units. _____

        e. Identify the Factors and their levels. _____

        f. How many treatments are there? _____

    3. A medical investigator studied the relationship between the response to three blood pressure lowering drug types for hypertensive males and females. The investigator selected 30 adult males and 30 adult females and randomly assigned 10 males and 10 females to each of the three drug types, yielding 60 total subjects.

        a. Is this an example of *experimental* or *observational* study? _____

        b. What study design was employed in this study? _____

        c. Identify the response variable. _____

        d. Identify the Experimental units. _____

        e. Identify the Factors and their levels. _____

        f. How many treatments are there? _____

- **The One-Factor-at-a-Time (OFAAT) Approach to Experimentation**

  In this approach, investigators vary only one factor at a time, holding all others constant, when attempting to understand the effect of a given set of factors on a particular outcome.

  - For example, to maximize sales in Example 1, we might be tempted to first fix price at a particular value such as 60 cents, and then determine which mode of advertising (radio or newspaper) is most effective. If this test reveals that newspaper advertising leads to higher sales, we would then run a second test in which the advertising mode is fixed at *newspaper*, and the three price levels are tested.
  - Deficiencies of the OFAACT approach:
    1. The OFAAT approach does not explore the entire space of treatment combinations, and important treatment combinations may therefore be missed.
    2. Interactions cannot be estimated.
    3. A full randomization is not possible for the OFAAT approach, because the experiment must be fielded in stages.
    4. The OFAAT approach is often more difficult to field logistically, because of the sequence of stages.

- **Advantages of Crossed, Multi-Factor Designs**

  1. Multi-factor studies are more *efficient* than OFAAT and provides *hidden* replication.
  2. It allows *assesment of interactions.*
  3. It can also strengthen the *validity of findings.*

- **Meaning of ANOVA Model Elements**

  1. Consider the example in the table below:

	Young	Middle	Old
Male	$\mu_{11} = 9$	$\mu_{12} = 11$	$\mu_{13} = 16$
Female	$\mu_{12} = 9$	$\mu_{22} = 11$	$\mu_{23} = 16$

     a. Main Age Effects:
     b. Main Gender Effects:
     c. Additive Factor Effects:

2. An example with Age and Gender effects, but with no interactions.

	Young	Middle	Old
Male	$\mu_{11} = 11$	$\mu_{12} = 13$	$\mu_{13} = 18$
Female	$\mu_{12} = 7$	$\mu_{22} = 9$	$\mu_{23} = 14$

3. An example with interacting factor effects.

	Young	Middle	Old
Male	$\mu_{11} = 9$	$\mu_{12} = 12$	$\mu_{13} = 18$
Female	$\mu_{12} = 9$	$\mu_{22} = 10$	$\mu_{23} = 14$

Interactions effects:

	Young	Middle	Old
Male			
Female			

4. An example with unimportant interactions.

	Young	Middle	Old
Male	$\mu_{11} = 9.75$	$\mu_{12} = 12$	$\mu_{13} = 17.25$
Female	$\mu_{12} = 8.25$	$\mu_{22} = 10$	$\mu_{23} = 14.75$

Interactions effects:

	Young	Middle	Old
Male			
Female			