Balanced Two-Way ANOVA

- **Two-Factor ANOVA Model:**

$$Y_{ijk} = \mu_{..} + \alpha_i + \beta_j + \alpha\beta_{ij} + \epsilon_{ijk}$$

where:

1. Y_{ijk} is the kth response under the ith level of factor A and jth level in factor B.
2. $\mu_{..}$ is the grand mean.
3. α_i is the effect of the ith level of factor A.
4. β_j is the effect of the jth level of factor B.
5. $\alpha\beta_{ij}$ is the interaction effect of the ith level of factor A and jth level of factor B.
6. ϵ_{ijk} are independent $N(0, \sigma^2)$.
7. $i = 1, \ldots, a$; $j = 1, \ldots, b$; $k = 1, \ldots, n$.

- Balanced Two-Way ANOVA Table with Fixed Factor Levels

ANOVA Table

Source	DF	Sum of Squares	Mean Square	F
Factor A	$a-1$	$SSA = \sum_{i=1}^{a} nb(\bar{Y}_{i..} - \bar{Y}_{...})^2$	$MSA = \frac{SSA}{a-1}$	$F_{\text{A}} = \frac{MSA}{MSE}$
Factor B	$b-1$	$SSB = \sum_{j=1}^{b} na(\bar{Y}_{.j.} - \bar{Y}_{...})^2$	$MSB = \frac{SSB}{b-1}$	$F_{\text{B}} = \frac{MSB}{MSE}$
AB Interaction	$(a-1)(b-1)$	$SSAB$	$MSAB = \frac{SSAB}{(a-1)(b-1)}$	$F_{\text{AB}} = \frac{MSAB}{MSE}$
Error	$ab(n-1)$	$SSE = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(Y_{ijk} - \bar{Y}_{ij.})^2$	$MSE = \frac{SSE}{ab(n-1)}$	
Total	$nab-1$	$SSTO = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n}(Y_{ijk} - \bar{Y}_{...})^2$		

Note:

1. $SSAB = \sum_{i=1}^{a}\sum_{j=1}^{b} n(\bar{Y}_{ij.} - \bar{Y}_{i..} - \bar{Y}_{.j.} + \bar{Y}_{...})^2$
2. $SSTR = SSA + SSB + SSAB$.
3. $SSTo = SSTR + SSE$.
4. $E(MSA) = \sigma^2 + bn\frac{\sum(\mu_{i.} - \mu_{..})^2}{a-1}$.
5. $E(MSB) = \sigma^2 + an\frac{\sum(\mu_{.j} - \mu_{..})^2}{b-1}$.
6. $E(MSAB) = \sigma^2 + n\frac{\sum\sum(\mu_{ij} - \mu_{i.} - \mu_{.j} + \mu_{..})^2}{(a-1)(b-1)}$.
7. $E(MSE) = \sigma^2$.

- **Castle Bakery Company Example**

  The Castle Bakery Company supplies wrapped Italian bread to a large number of supermarkets in a metropolitan area. A study was made of the effects of height of the shelf display (factor A: 1=bottom, 2=middle, 3=top) and the width of the shelf display (factor B: 1=regular, 2=wide) on sales of this bakery's bread during the study period (Y, measure in cases). Twelve supermarkets, similar in terms of sales volume and clientele, were utilized in the study. The treatments were assigned at random to two stores and the results are shown in the table below:

	Regular	Wide
Bottom	47	46
	43	40
Middle	62	67
	68	71
Tope	41	42
	39	46

```
data=read.csv("CastleBakery.csv",header=T)
attach(data)

A=factor(height)
B=factor(width)
result=aov(sales~A*B)
anova(result)
          Df Sum Sq Mean Sq F value    Pr(>F)
A          2   1544  772.00 74.7097 5.754e-05 ***
B          1     12   12.00  1.1613    0.3226
A:B        2     24   12.00  1.1613    0.3747
Residuals  6     62   10.33

tapply(sales,A,mean)
#  1  2  3
# 44 67 42
tapply(sales,B,mean)
#  1  2
# 50 52
```