## Analysis of Factor Effects

- **Factors Do Not Interact**

**1. Estimation of Factor Level Mean:**

Unbiased point estimators of $\mu_{i\cdot}$ and $\mu_{\cdot j}$ are:

$$\hat{\mu}_{i\cdot} = \bar{Y}_{i\cdot\cdot} \quad \text{and} \quad \hat{\mu}_{\cdot j} = \bar{Y}_{\cdot j\cdot}$$

Properties:

**a.** $E\{\bar{Y}_{i\cdot\cdot}\} = \mu_{i\cdot}$ and $E\{\bar{Y}_{\cdot j\cdot}\} = \mu_{\cdot j}$

**b.** $\sigma^2\{\bar{Y}_{i\cdot\cdot}\} = \dfrac{\sigma^2}{bn}$ and $\sigma^2\{\bar{Y}_{\cdot j\cdot}\} = \dfrac{\sigma^2}{an}$

**c.** $s^2\{\bar{Y}_{i\cdot\cdot}\} = \dfrac{MSE}{bn}$ and $s^2\{\bar{Y}_{\cdot j\cdot}\} = \dfrac{MSE}{an}$

**d.** $\dfrac{\bar{Y}_{i\cdot\cdot} - \mu_{i\cdot}}{s\{\bar{Y}_{i\cdot\cdot}\}} \sim t_{df=(n-1)ab}$ and $\dfrac{\bar{Y}_{\cdot j\cdot} - \mu_{\cdot j}}{s\{\bar{Y}_{\cdot j\cdot}\}} \sim t_{df=(n-1)ab}$

**e.** The $(1-\alpha)$ C.I. for $\mu_{i\cdot}$ is given by $\bar{Y}_{i\cdot\cdot} \pm t_{\frac{\alpha}{2}} s\{\bar{Y}_{i\cdot\cdot}\}$

**f.** The $(1-\alpha)$ C.I. for $\mu_{\cdot j}$ is given by $\bar{Y}_{\cdot j\cdot} \pm t_{\frac{\alpha}{2}} s\{\bar{Y}_{\cdot j\cdot}\}$

**2. Estimation of Contrast Level Means:**

An unbiased point estimator of the contrast $L = \sum_{i=1}^{a} c_i \mu_{i\cdot}$ is $\hat{L} = \sum_{i=1}^{a} c_i \bar{Y}_{i\cdot\cdot}$

**a.** $E\{\hat{L}\} = \sum_{i=1}^{r} c_i \mu_{i\cdot}$

**b.** $\sigma^2\{\hat{L}\} = \displaystyle\sum_{i=1}^{a} c_i^2 \frac{\sigma^2}{bn} = \frac{\sigma^2}{bn} \sum_{i=1}^{a} c_i^2$

**c.** $s^2\{\hat{L}\} = \dfrac{MSE}{bn} \displaystyle\sum_{i=1}^{a} c_i^2$

**d.** $\dfrac{\hat{L} - L}{s\{\hat{L}\}} \sim t_{df=(n-1)ab}$

**e.** The $(1-\alpha)$ C.I. for $L$ is given by $\hat{L} \pm t_{\frac{\alpha}{2}} s\{\hat{L}\}$

An unbiased point estimator of the contrast $L = \sum_{j=1}^{b} c_j \mu_{\cdot j}$ is $\hat{L} = \sum_{j=1}^{b} c_j \bar{Y}_{\cdot j\cdot}$

**a.** $E\{\hat{L}\} = \sum_{j=1}^{b} c_j \mu_{\cdot j}$

**b.** $\sigma^2\{\hat{L}\} = \displaystyle\sum_{j=1}^{b} c_j^2 \frac{\sigma^2}{an} = \frac{\sigma^2}{an} \sum_{j=1}^{b} c_j^2$

**c.** $s^2\{\hat{L}\} = \dfrac{MSE}{an} \displaystyle\sum_{j=1}^{b} c_j^2$

**d.** $\dfrac{\hat{L} - L}{s\{\hat{L}\}} \sim t_{df=(n-1)ab}$

**e.** The $(1-\alpha)$ C.I. for $L$ is given by $\hat{L} \pm t_{\frac{\alpha}{2}} s\{\hat{L}\}$

3. **Tukey Multiple Comparison Procedure:**
The Tukey multiple comparison confidence limits for all pairwise comparisons $D = \mu_{i\cdot} - \mu_{i'\cdot}$ with family confidence coefficient of at least $1-\alpha$ are as follows:
$$\hat{D} \pm Ts\{\hat{D}\}$$
where,
**a.** $\hat{D} = \bar{Y}_{i\cdot\cdot} - \bar{Y}_{i'\cdot\cdot}$
**b.** $s^2\{\hat{D}\} = \dfrac{2MSE}{bn}$
**c.** $T = \frac{1}{\sqrt{2}}q(1-\alpha; a, (n-1)ab)$
The Tukey multiple comparison confidence limits for all pairwise comparisons $D = \mu_{\cdot j} - \mu_{\cdot j'}$ with family confidence coefficient of at least $1-\alpha$ are as follows:
$$\hat{D} \pm Ts\{\hat{D}\}$$
where,
**a.** $\hat{D} = \bar{Y}_{\cdot j'\cdot} - \bar{Y}_{\cdot j'\cdot}$
**b.** $s^2\{\hat{D}\} = \dfrac{2MSE}{an}$
**c.** $T = \frac{1}{\sqrt{2}}q(1-\alpha; b, (n-1)ab)$

4. **Scheffe Multiple Comparison Procedure:**
When a large number of contrasts among the factor level mean $\mu_{i\cdot}$ or $\mu_{\cdot j}$ are of interest, the Scheffe method might be better. The Scheffe confidence limits for $L = \sum_{i=1}^{a} c_i \mu_{i\cdot}$ are:
$$\hat{L} \pm Ss\{\hat{L}\}$$
where,
$$S^2 = (a-1)F[1-\alpha; a-1, (n-1)ab]$$

- **Factors Interact**

When important interactions exist that cannot be make unimportant by a simple transformation, the analysis of factor effects generally must be based on the treatment means $\mu_{ij}$.

1. **Tukey Procedure.** The Tukey $1-\alpha$ multiple comparison confidence limits for all pairwise comparisons, $D = \mu_{ij} - \mu_{i'j'}$ are:
$$\hat{D} \pm Ts\{\hat{D}\}$$
where,
**a.** $\hat{D} = \bar{Y}_{ij\cdot} - \bar{Y}_{i'j'\cdot}$
**b.** $s^2\{\hat{D}\} = \dfrac{2MSE}{n}$
**c.** $T = \frac{1}{\sqrt{2}}q(1-\alpha; ab, (n-1)ab)$
2. **Bonferroni Procedure.** If the Bonferronni method is employed for a family of $g$ comparisons, the multiple in the confidence interval is replaced by:
$$B = t_{(1-\alpha/2g;(n-1)ab)}$$
3. **Scheffe Procedure.** The joint confidence limits for a contrast $L = \sum_{i=1}^{a}\sum_{j=1}^{b} c_{ij}\mu_{ij}$ are:
$$\hat{L} \pm Ss\{\hat{L}\}$$

where,

1. $\hat{L} = \sum_{i=1}^{a}\sum_{j=1}^{b} c_{ij}\bar{Y}_{ij\cdot}$
2. $s^2\{\hat{L}\} = \dfrac{MSE}{n}\displaystyle\sum_{i=1}^{a}\sum_{j=1}^{b} c_{ij}^2$
3. $S^2 = (ab-1)F[1-\alpha; ab-1, (n-1)ab]$