Randomized Complete Block Designs

- **Use of More than One Blocking Variable**

  1. Example 1:

Block	Characteristic of Experimental Units
1	Male, aged 20-29
2	Female, aged 20-29
3	Male, aged 30-39
4	Female, aged 30-39
etc.	etc.

  2. Example 2:

Block	Characteristic of Experimental Units
1	Observer 1, day 1
2	Observer 2, day 1
3	Observer 1, day 2
4	Observer 2, day 2
etc.	etc.

- **Use of More than One Replicate in Each Treatment within a Block.**

  When the nature of the interactions between *blocks* and *treatments* is of interest, more than one replicate is required in each treatment within a block. This design is called a ***Generalized Randomized Block Design***.

- **Generalized Randomized Block Design Model**

$$Y_{ijk} = \mu_{..} + \rho_i + \tau_j + (\rho\tau)_{ij} + \epsilon_{ijk} \quad \text{for } i = 1, \ldots, n_b; j = 1, \ldots, r; k = 1, \ldots, d$$

  where:

  1. Y_{ijk} is the kth response in the ith block and jth treatment.

  2. $\mu_{..}$ is a constant.

  3. ρ_i are constants for the block effects, subject to the restriction $\sum \rho_i = 0$.

  4. τ_j are constants for the treatment effects, subject to the restriction $\sum \tau_j = 0$.

  5. $(\rho\tau)_{ij}$ are constants for the interaction effects, subject to the restriction $\sum_i (\rho\tau)_{ij} = \sum_j (\rho\tau)_{ij} = 0$.

  6. ϵ_{ijk} are independent $N(0, \sigma^2)$.

- **ANOVA Table**

ANOVA Table

Source	DF	Sum of Squares	Mean Square	F
Blocks	$n_b - 1$	$SSBL = \sum_i r(\bar{Y}_{i.} - \bar{Y}_{..})^2$	$MSBL = \frac{SSBL}{n_b - 1}$	$F_{\text{BL}} = \frac{MSBL}{MSE}$
Treatments	$r - 1$	$SSTR = \sum_j n_b(\bar{Y}_{.j} - \bar{Y}_{..})^2$	$MSTR = \frac{SSTR}{r - 1}$	$F_{\text{TR}} = \frac{MSTR}{MSE}$
Interactions	$(n_b - 1)(r - 1)$	$SSBT$	$MSBT = \frac{SSBT}{(n_b - 1)(r - 1)}$	$F_{\text{BT}} = \frac{MSBTR}{MSE}$
Error	$n_b r(d - 1)$	SSE	$MSE = \frac{SSE}{n_b r(d - 1)}$	
Total	$dn_b r - 1$	$SSTO = \sum_i \sum_j \sum_k (Y_{ijk} - \bar{Y}_{...})^2$		

Note:

1. $SSE = \sum_{i=1}^{n_b}\sum_{j=1}^{r}\sum_{k=1}^{d}(Y_{ijk} - \bar{Y}_{ij\cdot})^2$
2. $SSBT = \sum_{i=1}^{n_b}\sum_{j=1}^{r} d(\bar{Y}_{ij\cdot} - \bar{Y}_{i\cdot\cdot} - \bar{Y}_{\cdot j\cdot} + \bar{Y}_{\cdots})^2$
3. $SSTO = SSBL + SSTR + SSBT + SSE.$
4. $E(MSBL) = \sigma^2 + rd\frac{\sum \rho_i^2}{n_b - 1}.$
5. $E(MSB) = \sigma^2 + n_b d\frac{\sum \tau_i^2}{r-1}.$
6. $E(MSBT) = \sigma^2 + d\frac{\sum\sum (\rho\tau)_{ij}^2}{(n_b-1)(r-1)}.$
7. $E(MSE) = \sigma^2.$

- **Example:** In a single-factor experiment in which the effects of distraction level (factor A: low or high) on the time required to complete a task were studied, using eight men and eight women. Four men were assigned at random to each treatment. The table below contains the results of the experiment.

Distraction	Male	Female
Low	12	3
	8	9
	7	5
	5	9
High	14	11
	16	9
	15	10
	13	14

- **R commands:**

```
> time=c(12,8,7,5,14,16,15,13,3,9,5,9,11,9,10,14)
> gender=gl(n=2,k=8) # n = no. of levels; k = no. of replications
> distraction=gl(2,4,length=16)
> interaction.plot(distraction,gender,time)

> result=aov(time~gender*distraction)
> anova(result)

Analysis of Variance Table
Response: time
                   Df Sum Sq Mean Sq F value    Pr(>F)
gender              1     25      25  4.1667 0.0638509 .
distraction         1    121     121 20.1667 0.0007385 ***
gender:distraction  1      4       4  0.6667 0.4301273
Residuals          12     72       6

> qqnorm(result$residuals)
> plot(result$fitted,result$residuals,xlab="Fitted values",ylab="Residuals")
```

- **Randomized Block Design for Factorial Treatments**

1. Example: Two-factor study in a randomized complete block design

	A_1		A_2	
Block	B_1	B_2	B_1	B_2
1	Y_{111}	Y_{112}	Y_{121}	Y_{122}
2	Y_{211}	Y_{212}	Y_{221}	Y_{222}
3	Y_{311}	Y_{312}	Y_{321}	Y_{322}

2. Model:

$$Y_{ijk} = \mu_{...} + \rho_i + \alpha_j + \beta_k + (\alpha\beta)_{jk} + \epsilon_{ijk} \quad \text{for } i = 1, \ldots, n_b; j = 1, \ldots, a; k = 1, \ldots, b$$

3. ANOVA Table:

ANOVA Table

Source	DF	Sum of Squares	Mean Square	F
Blocks	$n_b - 1$	$SSBL$	$MSBL = \frac{SSBL}{n_b - 1}$	$F_{\text{BL}} = \frac{MSBL}{MSE}$
Treatments	$r - 1$	$SSTR$	$MSTR = \frac{SSTR}{r - 1}$	$F_{\text{TR}} = \frac{MSTR}{MSE}$
Factor A	$a - 1$	SSA	$MSA = \frac{SSA}{a - 1}$	$F_{\text{A}} = \frac{MSA}{MSE}$
Factor B	$b - 1$	SSB	$MSB = \frac{SSB}{b - 1}$	$F_{\text{B}} = \frac{MSB}{MSE}$
AB Interactions	$(a-1)(b-1)$	$SSAB$	$MSAB = \frac{SSAB}{(a-1)(b-1)}$	$F_{AB} = \frac{MSAB}{MSE}$
Error	$(n_b - 1)(r - 1)$	SSE	$MSE = \frac{SSE}{(n_b - 1)(r - 1)}$	
Total	$n_b r - 1$	$SSTO$		

Note:

a. $SSTR = SSA + SSB + SSAB$.

b. $SSTO = SSBL + SSTR + SSE$.

4. R commands:

```
> result=aov(response~block+factorA+factorB+factorA:factorB)
> anova(result)
```