Two-Factor Studies with Unequal Sample Sizes

- **Growth Hormone Example.** Synthetic growth hormone was administered at a clinical research center to growth hormone deficient, short children who had not yet reached puberty. The investigator was interested in the effects of a child's gender (factor A) and bone development (factor B) on the rate of growth induced by hormone administration. A child's bone development was classified into one of three categories: severely depressed, moderately depressed, mildly depressed. Three children were randomly selected for each gender-bone development group. The response variable of interest was the difference between the growth rate during growth hormone treatment and normal growth rate prior to the treatment, expressed in centimeters per month. Four of the 18 children were unable to complete the year-long study, thus creating unequal treatment sample sizes. The results are shown in the table below:

	Severely	Moderately	Mildly
Male	1.4	2.1	0.7
	2.4	1.7	1.1
	2.2		
Female	2.4	2.5	0.5
		1.8	0.9
		2	1.3

- **Model:** $Y_{ijk} = \mu_{\cdot\cdot} + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \epsilon_{ijk}$
- **SPSS Results** (Analyze --→ General Linear Model --→ Univariate)

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Tests of Between-Subjects Effects
Dependent Variable: Growth
Source          Type III Sum of Squares     df  Mean Square       F   Sig.
Gender          0.120                       1   0.120         0.738  0.415
Bone            4.190                       2   2.095        12.891  0.003
Gender*Bone     0.075                       2   0.038         0.232  0.798
Error           1.300                       8   0.163
Total           43.560                      14
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1. Using a 5% significance level, test if there is an interaction effect between gender and bone development.

2. Using a 5% significance level, test if gender has an effect on growth rate.

3. Using a 5% significance level, test if bone development has an effect on growth rate.

4. Construct a 95% confidence interval for $\mu_{1\cdot}$.

5. Using Bonferroni adjustment, construct 95% family confidence intervals for $D_1 = \mu_{\cdot 1} - \mu_{\cdot 3}$ and $D_2 = \mu_{\cdot 2} - \mu_{\cdot 3}$. Determine the multiple if you used the Tukey or the Scheffe method instead.

6. Construct a 95% confidence interval for the contrast $L = \dfrac{\mu_{\cdot 1} + \mu_{\cdot 2}}{2} - \mu_{\cdot 3}$.

7. Construct a 95% confidence interval for $L = 0.2\mu_{\cdot 1} + 0.3\mu_{\cdot 2} + 0.5\mu_{\cdot 3}$.

8. Construct a 95% confidence interval for μ_{11}.

9. Using Bonferroni adjustment, construct 95% family confidence intervals for $D_1 = \mu_{12} - \mu_{22}$ and $D_2 = \mu_{13} - \mu_{23}$. Determine the multiple if you used the Tukey or the Scheffe method instead.

10. Construct a 95% confidence interval for the contrast $L = \dfrac{\mu_{11} + \mu_{12}}{2} - \dfrac{\mu_{21} + \mu_{22}}{2}$.