# Multi-Factor Studies

- **Analysis of Factor Effects when Factors Do Not Interact**

  **1.** Estimation of Factor Level Mean.

  **a.** $\hat{\mu}_{i\cdot\cdot} = \bar{Y}_{i\cdot\cdot\cdot}$

  **b.** $s^2\{\bar{Y}_{i\cdot\cdot\cdot}\} = \frac{MSE}{nbc}$

  **c.** The $(1-\alpha)100\%$ confidence interval: $\bar{Y}_{i\cdot\cdot\cdot} \pm t_{[1-\alpha/2;(n-1)abc]} s\{\bar{Y}_{i\cdot\cdot\cdot}\}$

  **2.** Inferences for Contrast of Factor Level Means

  **a.** If $L = \sum_i c_i \mu_{i\cdot\cdot}$, where $\sum_i c_i = 0$, then $\hat{L} = \sum_i c_i \bar{Y}_{i\cdot\cdot\cdot}$

  **b.** $s^2\{\hat{L}\} = \frac{MSE}{nbc} \sum_i c_i^2$

  **c.** The $(1-\alpha)100\%$ confidence interval: $\hat{L} \pm t_{[1-\alpha/2;(n-1)abc]} s\{\hat{L}\}$

  **3.** Multiple Contrasts of Factor Level Means

| Procedure | Multiple |
|---|---|
| Tukey | $T = \frac{1}{\sqrt{2}} q[1-\alpha; a, (n-1)abc]$ |
| Scheffe | $S^2 = (a-1)F(1-\alpha; a-1, (n-1)abc)$ |
| Bonferroni | $B = t[1-\alpha/(2*g); (n-1)abc]$ |

- **Analysis of Factor Effects with Multiple Two-Factor Interactions**

  **1.** Estimation of Treatment Mean.

  **a.** $\hat{\mu}_{ijk} = \bar{Y}_{ijk\cdot}$

  **b.** $s^2\{\bar{Y}_{ijk\cdot}\} = \frac{MSE}{n}$

  **c.** The $(1-\alpha)100\%$ confidence interval: $\bar{Y}_{ijk\cdot} \pm t_{[1-\alpha/2;(n-1)abc]} s\{\bar{Y}_{ijk\cdot}\}$

  **2.** Inferences for Contrast of Treatment Means

  **a.** If $L = \sum\sum\sum c_{ijk}\mu_{ijk}$, where $\sum\sum\sum c_{ijk} = 0$, then $\hat{L} = \sum\sum\sum c_{ijk}\bar{Y}_{ijk\cdot}$

  **b.** $s^2\{\hat{L}\} = \frac{MSE}{n} \sum\sum\sum c_{ijk}^2$

  **c.** The $(1-\alpha)100\%$ confidence interval: $\hat{L} \pm t_{[1-\alpha/2;(n-1)abc]} s\{\hat{L}\}$

- **Analysis of Factor Effects with Single Two-Factor Interaction**

  **1.** Suppose only $BC$ interactions are present in a three-factor study.

  **a.** If $L = \sum_k \sum_j c_{jk}\mu_{\cdot jk}$, where $\sum_k \sum_j c_{jk} = 0$, then $\hat{L} = \sum_k \sum_j c_{jk}\bar{Y}_{\cdot jk\cdot}$

  **b.** $s^2\{\hat{L}\} = \frac{MSE}{na} \sum_k \sum_j c_{jk}^2$

  **c.** The $(1-\alpha)100\%$ confidence interval: $\hat{L} \pm t_{[1-\alpha/2;(n-1)abc]} s\{\hat{L}\}$

  **2.** Multiple Contrasts of Factor Level Means

| Procedure | Multiple |
|---|---|
| Tukey | $T = \frac{1}{\sqrt{2}} q[1-\alpha; abc, (n-1)abc]$ |
| Scheffe | $S^2 = (abc-1)F(1-\alpha; abc-1, (n-1)abc)$ |
| Bonferroni | $B = t[1-\alpha/(2*g); (n-1)abc]$ |

- **Stress Test Example**

  The effects of gender of subject (factor A: male or female), body fat of subject (measured in percent, factor B: low or high), and smoking history of subject(factor C: light or heavy) on exercise tolerance ($Y$) were studied in a small-scale investigation of persons 25 to 35 years old. Exercise tolerance was measured in minutes until fatigue occurs while the subject is performing on a bicycle apparatus. Three subjects for each gender-body fat-smoking history group were given the exercise tolerance stress test. The results are shown in the table below:

| | Light | Heavy |
|---|---|---|
| Low fat | | |
| Male | 24.1 | 17.6 |
| | 29.2 | 18.8 |
| | 24.6 | 23.2 |
| Female | 20.0 | 14.8 |
| | 21.9 | 10.3 |
| | 17.6 | 11.3 |
| High fat | | |
| Male | 14.6 | 14.9 |
| | 15.3 | 20.4 |
| | 12.3 | 12.8 |
| Female | 16.1 | 10.1 |
| | 9.3 | 14.4 |
| | 10.8 | 6.1 |

- **R commands:**

```
data=read.csv("StressTest.csv",header=T)
attach(data)

A=factor(gender)
B=factor(fat)
C=factor(smoking)
result=aov(tolerance~A*B*C)
anova(result)
Response: tolerance
          Df  Sum Sq Mean Sq F value    Pr(>F)
A          1 176.584 176.584 18.9155 0.0004971 ***
B          1 242.570 242.570 25.9839 0.0001076 ***
C          1  70.384  70.384  7.5394 0.0143574 *
A:B        1  13.650  13.650  1.4622 0.2441432
A:C        1  11.070  11.070  1.1859 0.2922989
B:C        1  72.454  72.454  7.7612 0.0132205 *
A:B:C      1   1.870   1.870  0.2004 0.6604336
Residuals 16 149.367   9.335

means.ABC=tapply(tolerance,A:B:C,mean)
   1:1:1    1:1:2    1:2:1    1:2:2    2:1:1    2:1:2    2:2:1    2:2:2
25.96667 19.86667 14.06667 16.03333 19.83333 12.13333 12.06667 10.20000

means.A=tapply(tolerance,A,mean)
       1        2
18.98333 13.55833
```

- **Practice.** To study the nature of the $BC$ interaction effects in the stress test example, the researcher wished to estimate separately, for persons with high and low body fat, the difference in mean fatigue time for light smokers and heavy smokers. The desired contrasts are:

$$L_1 = \mu_{\cdot 11} - \mu_{\cdot 12} \quad \text{and} \quad L_2 = \mu_{\cdot 21} - \mu_{\cdot 22}$$

  In addition, a single comparison between the factor level means for factor A is sufficient to analyze the factor A main effects since factor A has only two levels. The contrast of interest is

$$L_3 = \mu_{1\cdot\cdot} - \mu_{2\cdot\cdot}$$

  Use $\alpha = 0.05$ and the Bonferroni adjustment.