Nested Designs

- **Training School Example**. A large manufacturing company operates three regional training schools for mechanics, one in each of its operating districts. The schools have two instructors each, who teach classes of about 15 mechanics in three-week sessions. The company was concerned about the effect of school (factor A) and instructor (factor B) on the learning achieved. To investigate these effects, classes in each district were formed in the usual way and then randomly assigned to one of the the two instructors in the school. This was done for two sessions, and at the end of each session a suitable summary measure of learning for the class was obtained. The results are shown below:

School	Instructor 1	Instructor 2
Atlanta	25	14
	29	11
Chicago	11	22
	6	18
San Francisco	17	5
	20	2

- **R commands:**

```
response=c(25,29,14,11,11,6,22,18,17,20,5,2)
school=gl(3,4)
instructor=gl(2,2,12)

result=aov(response~school/instructor)
anova(result)
Response: response
                  Df Sum Sq Mean Sq F value   Pr(>F)
school             2  156.5   78.25  11.179 0.009473 **
school:instructor  3  567.5  189.17  27.024 0.000697 ***
Residuals          6   42.0    7.00
```

- **Estimation of Contrasts**: For fixed factor A.

  1. $L = \sum c_i \mu_{i\cdot}$, where $\sum c_i = 0$

  2. $\hat{L} = \sum c_i \bar{Y}_{i\cdot\cdot}$

  3. $s^2\{\hat{L}\} = \sum c_i^2 s^2\{\bar{Y}_{i\cdot\cdot}\}$

  4. The $(1-\alpha)100\%$ confidence interval: $\hat{L} \pm t_{[1-\alpha/2;df]} s\{\hat{L}\}$

  5. *Example.* In the Training School example, suppose it was desired to estimate the difference in mean learning score for the Atlanta and Chicago schools with a 95% confidence interval.

- **Multiple Comparison Procedures**. Multiple comparison procedures that we studied before are applicable for the main effects of the fixed factor.

  1. Tukey: $T = \frac{1}{\sqrt{2}} q[1-\alpha; a, ab(n-1)]$
  2. Scheffe: $S^2 = (a-1)f[1-\alpha; (a-1), ab(n-1)]$
  3. Bonferroni: $B = t[1-\alpha/(2*g), ab(n-1)]$
  4. *Example.* In the Training School example, suppose it was desired to do all pairwise comparisons of the three schools with family confidence coefficient of 0.90.

- **R commands:**

```
> TukeyHSD(result,"school",conf.level=.90)
     diff        lwr        upr     p adj
2-1 -5.50 -10.207283 -0.7927168 0.0586129
3-1 -8.75 -13.457283 -4.0427168 0.0081224
3-2 -3.25  -7.957283  1.4572832 0.2676596
```

- **Estimation of Treatment Means (μ_{ij})**: For fixed factors A and B.

  1. $\hat{\mu}_{ij} = \bar{Y}_{ij\cdot}$
  2. $s^2\{\bar{Y}_{ij\cdot}\} = \frac{MSE}{n}$
  3. The $(1-\alpha)100\%$ confidence interval: $\bar{Y}_{ij\cdot} \pm t_{[1-\alpha/2; ab(n-1)]} s\{\bar{Y}_{ij\cdot}\}$
  4. *Example.* In the Training School example, suppose it was desired to estimate the mean learning score for the first instructor in Atlanta school with a 95% confidence interval.

- **Estimation of Contrasts of Treatment Means**: For fixed factors A and B.

  1. $L = \sum c_j \mu_{ij}$, where $\sum c_j = 0$

  2. $\hat{L} = \sum c_j \bar{Y}_{ij\cdot}$

  3. $s^2\{\hat{L}\} = \frac{MSE}{n} \sum c_j^2$

  4. The $(1-\alpha)100\%$ confidence interval: $\hat{L} \pm t_{[1-\alpha/2;ab(n-1)]} s\{\hat{L}\}$

  5. *Example.* In the Training School example, suppose it was desired to compare the mean scores for the two instructors in each with 90% family confidence intervals using the Bonferroni adjustment.

- **Estimation of Overall Mean** ($\mu_{\cdot\cdot}$):

  1. $\hat{\mu}_{\cdot\cdot} = \bar{Y}_{\cdot\cdot\cdot}$

  2. $s^2\{\bar{Y}_{\cdot\cdot\cdot}\} = \frac{MSE}{abn}$ with $df = ab(n-1)$ when A and B are fixed.

  3. $s^2\{\bar{Y}_{\cdot\cdot\cdot}\} = \frac{MSA}{abn}$ with $df = a-1$ when A and B are random.

  4. $s^2\{\bar{Y}_{\cdot\cdot\cdot}\} = \frac{MSB(A)}{abn}$ with $df = a(b-1)$ when A is fixed and B is random.

  5. The $(1-\alpha)100\%$ confidence interval: $\bar{Y}_{\cdot\cdot\cdot} \pm t_{[1-\alpha/2;df]} s\{\bar{Y}_{\cdot\cdot\cdot}\}$

  6. *Example.* In the Training School example, suppose it was desired to estimate the overall mean learning score for all training schools and all instructors in these schools with a 95% confidence interval.

- **Estimation of Variance Components**. When both A and B are random.

  1. Note $\sigma_\alpha^2 = \dfrac{E(MSA) - E(MSB(A))}{bn}$

  2. An unbiased estimator of σ_α^2 is $s_\alpha^2 = \dfrac{MSA - MSB(A)}{bn}$

  3. *Satterthwaite Procedure*. An approximate $(1-\alpha)100\%$ confidence interval for σ_α^2 is

$$\frac{(df)s_\alpha^2}{\chi^2[1-\alpha/2; df]} \quad \text{and} \quad \frac{(df)s_\alpha^2}{\chi^2[\alpha/2; df]}$$

  where,

$$df = \frac{(s_\alpha^2)^2}{\frac{(MSA/bn)^2}{a-1} + \frac{(MSB(A)/bn)^2}{a(b-1)}}$$