Repeated Measures

Repeated measures (RM) designs utilize the same subject (person, store, plant, test market, etc.) for each of the treatments under study. The subject therefore serves as a block, and the experimental units within a block may be viewed as the different occasions when a treatment is applied to the subject.

- **Examples of Repeated Measures Designs**

  1. Fifteen test markets are to be used to study each of two different advertising campaigns. In each test market, the order of the two campaigns will be randomized, with sufficient time lapse between two campaigns so that the effects of the initial campaign will not carry over into the second campaign.

  2. Two hundred persons who have persistent migraine headaches are each to be given two different drugs and a placebo, for two weeks each, with order of the drugs randomized for each person. The subjects in the study are the persons with migraine headaches.

  3. In a weight loss study, 100 overweight persons are to be given the same diet and their weights measured at the end of each week for 12 weeks to assess the weight loss over time.

- **Advantages of Repeated Measures Design**

  1. It provides good precision for comparing treatments because all sources of variability between subjects are excluded from the experimental error.

  2. It economizes on subjects.

  3. It provides a way to see the effects of a treatment on a subject over time.

- **(Potential) Disadvantages of Repeated Measures Design**. Types of Interference

  1. *Order Effect* - subjects may tend to give higher (or lower) ratings for the last treatment than the first treatment. The *order effect* could be minimize by randomization of treatment orders.

  2. *Carryover Effect* - subjects may rate a treatment higher (or lower) depending on the previous treatment. The *carryover effect* could be minimize by allowing sufficient time between treatments. Use of *Latin square designs* and *crossover designs* are helpful to this end.

- **Single-Factor Experiments with Repeated Measures on all Treatments**. This design is the same as the randomized block design with random block effects.

$$Y_{ij} = \mu_{\cdot\cdot} + \rho_i + \tau_j + \epsilon_{ij}, \qquad \text{for } i = 1, \ldots, s \text{ and } j = 1, \ldots, r$$

  where:

  1. Y_{ij} is the response in the ith subject and jth treatment.

  2. $\mu_{\cdot\cdot}$ is a constant.

  3. ρ_i are independent $N(0, \sigma_\rho^2)$.

  4. τ_j are constants subject to $\sum_{j=1}^{r} \tau_j = 0$.

  5. ϵ_{ij} are independent $N(0, \sigma^2)$.

  6. ρ_i and ϵ_{ij} are independent.

- **ANOVA Table**

ANOVA Table

Source	DF	Sum of Squares	Mean Square	$E(MS)$
Subjects	$s-1$	$SSS = \sum_i r(\bar{Y}_{i\cdot} - \bar{Y}_{\cdot\cdot})^2$	$MSS = \frac{SSS}{s-1}$	$\sigma^2 + r\sigma_\rho^2$
Treatments	$r-1$	$SSTR = \sum_j s(\bar{Y}_{\cdot j} - \bar{Y}_{\cdot\cdot})^2$	$MSTR = \frac{SSTR}{r-1}$	$\sigma^2 + s\frac{\sum \tau_j^2}{r-1}$
Error	$(s-1)(r-1)$	$SSE = \sum_i \sum_j (Y_{ij} - \bar{Y}_{i\cdot} - \bar{Y}_{\cdot j} + \bar{Y}_{\cdot\cdot})^2$	$MSE = \frac{SSE}{(s-1)(r-1)}$	σ^2
Total	$sr-1$	$SSTO = \sum_i \sum_j (Y_{ij} - \bar{Y}_{\cdot\cdot})^2$		

Note: $SSTO = SSTR + SSS + SSE$.

- **Example: Wine-Judging.** In a wine-judging competition, four Chardonnay wines of the same vintage were judged by six experience judges. Each judge tasted the wines in a blind fashion, i.e. without knowing their identities. The order of the wine presentation was randomized independently for each judge. To reduce carryover and other interference effects, the judges did not drink the wines and rinsed their mouths thoroughly between tastings. Each wine was scored on a 40-point scale; the higher the score, the greater is the excellence of the wine. The data are shown in the table below:

Judge	Wine 1	Wine 2	Wine 3	Wine 4
1	20	24	28	28
2	15	18	23	24
3	18	19	24	23
4	26	26	30	30
5	22	24	28	26
6	19	21	27	25

- **R commands:**

```
> response=c(20,24,28,28,15,18,23,24,18,19,24,23,26,26,30,30,22,24,28,26,19,21,27,25)
> subjects=gl(n=6,k=4) # n = no. of levels; k = no. of replications
> treatment=gl(4,1,length=24)
> data=data.frame(rating=response,judge=subjects,wine=treatment)
> interaction.plot(treatment,subjects,response)

> result=aov(response~subjects+treatment)
> anova(result)
Response: response
          Df Sum Sq Mean Sq F value    Pr(>F)
subjects   5 173.33  34.667    32.5 1.549e-07 ***
treatment  3 184.00  61.333    57.5 1.854e-08 ***
Residuals 15  16.00   1.067

> qqnorm(result$residuals)
> plot(result$fitted,result$residuals,xlab="Fitted values",ylab="Residuals")

> tapply(response,treatment,mean)
       1        2        3        4
20.00000 22.00000 26.66667 26.00000

> TukeyHSD(result,"treatment",conf.level=.95)
$treatment
          diff        lwr      upr     p adj
2-1  2.0000000  0.2814186 3.718581 0.0202300
3-1  6.6666667  4.9480852 8.385248 0.0000001
4-1  6.0000000  4.2814186 7.718581 0.0000003
3-2  4.6666667  2.9480852 6.385248 0.0000061
4-2  4.0000000  2.2814186 5.718581 0.0000375
4-3 -0.6666667 -2.3852481 1.051915 0.6844296
```

- **Analysis of Treatment Effects**

  1. Single comparison: $t = qt(1-\alpha/2, df = (s-1)*(r-1))$
  2. Tukey procedure for all pairwise comparison: $T = qtukey(1-\alpha, r, df = (s-1)*(r-1))$
  3. Scheffe procedure: $S = sqrt((r-1)*qf(1-\alpha, df1 = r-1, df2 = (s-1)*(r-1))$
  4. Bonferroni procedure: $B = qt(1-\alpha/(2*g), df = (s-1)*(r-1))$

- **Two-Factor Experiments with Repeated Measures on One Factor**

$$Y_{ijk} = \mu_{\cdots} + \rho_{i(j)} + \alpha_j + \beta_k + (\alpha\beta)_{jk} + \epsilon_{ijk}$$

  where:

  1. $\mu_{\cdots}$ is a constant.
  2. $\rho_{i(j)}$ are independent $N(0, \sigma_\rho^2)$.
  3. α_j are constants subject to $\sum_j \alpha_j = 0$.
  4. β_k are constants subject to $\sum_k \beta_k = 0$.
  5. $(\alpha\beta)_{jk}$ are constants subject to $\sum_j (\alpha\beta)_{jk} = 0$ for all k and $\sum_k (\alpha\beta)_{jk} = 0$ for all j.
  6. ϵ_{ij} are independent $N(0, \sigma^2)$.
  7. $\rho_{i(j)}$ and ϵ_{ij} are independent.
  8. $i = 1, \ldots, s; j = 1, \ldots, a; k = 1, \ldots, b$

- **ANOVA Table for Two-Factor Experiment with Repeated Measures on Factor B**

Source of Variation	DF	Sum of Squares	Mean Square	E(MS)
Factor A	$a-1$	SSA	$MSA = \frac{SSA}{a-1}$	$\sigma^2 + b\sigma_\rho^2 + bs\frac{\sum_j \alpha_j^2}{a-1}$
Factor B	$b-1$	SSB	$MSB = \frac{SSB}{b-1}$	$\sigma^2 + as\frac{\sum_k \beta_k^2}{b-1}$
AB interactions	$(a-1)(b-1)$	$SSAB$	$MSAB = \frac{SSAB}{(a-1)(b-1)}$	$\sigma^2 + s\frac{\sum_j \sum_k (\alpha\beta)_{jk}}{(a-1)(b-1)}$
Subjects(within factor A)	$a(s-1)$	$SSS(A)$	$MSS(A) = \frac{SSS(A)}{a(s-1)}$	$\sigma^2 + b\sigma_\rho^2$
Error	$a(s-1)(b-1)$	SSE	$MSE = \frac{SSE}{a(s-1)(n-1)}$	σ^2
Total	$abs-1$	$SSTO$		

- **R commands:**

```
ad1=c(958,1047,933,1005,1122,986,351,436,339,549,632,512,730,784,707)
ad2=c(780,897,718,229,275,202,883,964,817,624,695,599,375,436,351)
response=c(ad1,ad2)
time=gl(3,1,30)
ad=gl(2,15,30)
subject=gl(10,3)

result=aov(response~ad/subject+ad*time)
anova(result)
Response: response
           Df  Sum Sq Mean Sq  F value    Pr(>F)
ad          1  168151  168151 469.738* 2.762e-13 ***
time        2   67073   33537  93.6862 1.468e-09 ***
ad:subject  8 1833681  229210 640.3113 < 2.2e-16 ***
ad:time     2     391     196   0.5468    0.5892
Residuals  16    5727     358

F_ad=168151/229210          #F_ad=MS(ad)/MS(ad:subject) = 0.7336111
p_value_ad=1-pf(0.7336,1,8) #p_value_ad=0.4166
```

Take note that the $F-$value for ‘ad’ in the anova(result) is not correct.