Multinomial Distribution

- **Definition 5.11**. A *multinomial* experiment possesses the following properties:
  1. The experiment consists of n identical trials.
  2. The outcome of each trial falls into one of k classes or cells.
  3. The probability that the outcome of a single trial falls into cell i, is p_i, $i = 1, 2, \ldots, k$ and remains the same from trial to trial. Notice that $p_1 + p_2 + \ldots + p_k = 1$.
  4. The trials are independent.
  5. The random variables of interest are $Y_1, Y_2, \ldots, Y_k$, where Y_i equals the number of trials for which the outcome falls into cell i. Notice that $Y_1 + Y_2 + \ldots + Y_k = n$.

- **Definition 5.12**. Assume that $p_1, p_2, \ldots, p_k$ are such that $\sum_{i=1}^{k} p_i = 1$, and $p_i > 0$ for $i = 1, 2, \ldots, k$. The random variables $Y_1, Y_2, \ldots, Y_k$, are said to have a *multinomial distribution* with parameters n and $p_1, p_2, \ldots, p_k$ if the joint probability function of $Y_1, Y_2, \ldots, Y_k$ is given by

$$p(u_1, y_2, \ldots, y_k) = \frac{n!}{y_1! y_2! \cdots y_k!} p_1^{y_1} p_2^{y_2} \cdots p_k^{y_k},$$

  where, for each i, $y_i = 0, 1, 2, \ldots, n$ and $\sum_{i=1}^{k} y_i = n$.

- **Theorem 5.13**. If $Y_1, Y_2, \ldots, Y_k$, have a multinomial distribution with parameters n and $p_1, p_2, \ldots, p_k$, then
  1. $E(Y_i) = np_i$, and $V(Y_i) = np_i(1 - p_i)$
  2. $Cov(Y_s, Y_t) = -np_s p_t$, if $s \neq t$.

Bivariabte Normal Distribution

- **Definition**. Y_1 and Y_2 are said to have a *bivariate normal distribution* with parameters $\mu_1, \mu_2, \sigma_1, \sigma_2, \rho$ if the joint density of Y_1 and Y_2 is given by

$$f(y_1, y_2) = \frac{e^{-Q/2}}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}}, \qquad -\infty < y_1, y_2 < \infty$$

  where

$$Q = \frac{1}{1-\rho^2}\left[\frac{(y_1-\mu_1)^2}{\sigma_1^2} - 2\rho\frac{(y_1-\mu_1)(y_2-\mu_2)}{\sigma_1\sigma_2} + \frac{(y_2-\mu_2)^2}{\sigma_2^2}\right].$$

Conditional Expectations

- **Definition 5.13**. If Y_1 and Y_2 are any two random variables, the *conditional expectation* of $g(Y_1)$, given that $Y_2 = y_2$, is defined to be

$$E(g(Y_1)|Y_2 = y_2) = \int_{-\infty}^{\infty} g(y_1) f(y_1|y_2)\, dy_1$$

if Y_1 and Y_2 are jointly continuous and

$$E(g(Y_1)|Y_2 = y_2) = \sum_{y_1} g(y_1) f(y_1|y_2)$$

if Y_1 and Y_2 are jointly discrete.

- **Theorem 5.14**. Let Y_1 and Y_2 denote random variables. Then

$$E(Y_1) = E[E(Y_1|Y_2)]$$

where, on the right-hand side, the inside expectation is with respect to the conditional distribution of Y_1 given Y_2, and the outside expectation is with respect to the distribution of Y_2.

- **Theorem 5.15**. Let Y_1 and Y_2 denote random variables. Then

$$V(Y_1) = E[V(Y_1|Y_2)] + V[E(Y_1|Y_2)].$$

- **Recommended problems:**
Section 5.9: (pp. 282-283) # 119, 121, 123, 125;
Section 5.10: (pp. 284-285) # 128, 131;
Section 5.11: (pp. 289-290) # 133, 135, 139, 141.