Probability

1. **Definition 2.6** (By Kolmogorov) Suppose S is a sample space associated with an experiment. To every event A in S (A is a subset of S) we assign a number, $P(A)$, called the *probability* of A, so that the following axioms hold:

   a. Axiom 1: $P(A) \geq 0$.

   b. Axiom 2: $P(S) = 1$.

   c. Axiom 3: If $A_1, A_2, A_3, \ldots$ form a sequence of pairwise mutually exclusive events in S (that is, $A_i \cap A_j = \emptyset$ if $i \neq j$), then $P(A_1 \cup A_2 \cup A_3 \cup \ldots) = \sum_{i=1}^{\infty} P(A_i)$.

2. **Properties.**(Prove all)

   a. $P(\phi) = 0$.

   b. (Finite Additivity) If $A_1, A_2, \ldots, A_k$ are pairwise mutually exclusive events, then $P(A_1 \cup A_2 \cup \ldots \cup A_k) = \sum_{i=1}^{k} P(A_i)$.

   c. If $A \subseteq B$, then $P(A) \leq P(B)$.

   d. *Probability of the Union* [Theorem 2.6]: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
   If A and B are *mutually exclusive*, then $A \cap B$ is empty and $P(A \cap B) = 0$.

   e. *Probability of the Compliment* [Theorem 2.7]: $P(A^c) = 1 - P(A)$.

3. **Practice:**

   a. In a small town of 2000 people, there are 800 males, 700 of whom are employed. If a total 250 people are unemployed in this town, find the probability that a randomly selected person is

   i. a male?

   ii. an unemployed?

   iii. a male and unemployed (an unemployed male)?

   iv. male or unemployed?

   v. male given the person is unemployed?

   vi. unemployed given the person is a male?

   b. Suppose that in a sample of 500 college students, 350 drink alcohol, 200 smoke, and 150 drink and smoke. If a student is chosen at random from this sample, let D represent the event of selecting someone who drinks alcohol, and let S represent the event of selecting someone who smokes. What is the probability that the randomly selected student

   i. drinks alcohol but does not smoke?

   ii. is engaged in at least one of these two habits?

   iii. smokes given that he/she drinks?

4. **Conditional Probability.** [Definition 2.9] When $P(B) > 0$, the conditional probability of A given B is

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{P(A \cap B)}{P(B)}.$$

5. **The Multiplicative Law of Probability.** [Theorem 2.5] The probability of the intersection of two events A and B, $P(A \cap B) = P(A)P(B|A) = P(B)P(A|B)$

6. **Independence:** Events A and B are said to be *independent* if any one of the following holds:
   (i) $P(A|B) = P(A)$, (ii) $P(B|A) = P(B)$, (iii) $P(A \cap B) = P(A)P(B)$.

   Question 1: In practice problem (b), are events D and S independent? Explain your answer.

   Question 2: In practice problem (b), are events D and S mutually exclusive? Explain your answer.

   Question 3: If 5% of the products produced by a certain machine have cosmetic flaws, what is the probability that two randomly selected products produced by this machine both have cosmetic flaws?

7. **Tree Diagram:** A chain of video store sells three different brands of Blu-Ray players (BRP). Of its BRP sales, 50% are brand 1 (the least expensive), 30% are brand 2, and 20% are brand 3. Each manufacturer offers a 1-year warranty on parts and labor. It is known that 25% of brand 1's BRP require warranty repair work, whereas the corresponding percentages for brands 2 and 3 are 20% and 10%, respectively.

   a. What is the probability that a randomly selected purchaser has bought a brand 1 BRP that will need repair while under warranty?

   b. What is the probability that a randomly selected purchaser has a BRP that will need repair while under warranty?

   c. If a customer returns to the store with a BRP that needs warranty repair work, what is the probability that it is a brand 1 BRP?

8. **Partition.** [Definition 2.11] For some positive integer k, let the sets $B_1, B_2, \ldots, B_k$ be such that

   a. $S = B_1 \cup B_2 \cup \cdots \cup B_k$.

   b. $B_i \cap B_j = \emptyset$ for $i \neq j$.

   Then the collection of sets $\{B_1, B_2, \ldots, B_k\}$ is said to be a *partition* of S.

9. **Total Probability:** Assume that $\{B_1, B_2, \ldots, B_k\}$ is a partition of S such the $P(B_i) > 0$, for $i = 1, 2, \ldots, k$. Then for any event A, $P(A) = \sum_{i=1}^{k} P(A|B_i)P(B_i)$

10. **Bayes Rule:** Assume that $\{B_1, B_2, \ldots, B_k\}$ is a partition of S such the $P(B_i) > 0$, for $i = 1, 2, \ldots, k$. Then

$$P(B_j|A) = \frac{P(A|B_j)P(B_j)}{\sum_{i=1}^{k} P(A|B_i)P(B_i)}.$$

   Practice: Online chat rooms are dominated by the young. Teens are the biggest users. If we look only at adult Internet users (age 18 and over), 47% of the 18 to 29 age group chat, as do 21% of those aged 30 to 49 and just 7% of those 50 and over. It is known that 29% of adult Internet users are age 18 to 29, another 47% are 30 to 49 years old, and the remaining 24% are age 50 and over.

   a. the person is at least 50 years old?

   b. the person chats online given he/she is at least 50 years old?

   c. the person is at least 50 years old and chats online?

   d. the person chats online?

   e. the person is at least 50 years old given that the person chats online?

11. **Recommended Problems:**
    Section 2.7: (pp. 54 - 57) 71, 77, 79, 81, 83.
    Section 2.8: (pp. 59 - 62) 85, 91, 93, 95, 97, 101, 105.
    Section 2.9: (pp. 68 - 69) 111, 115, 121.
    Section 2.10: (pp. 72 -75) 125, 129, 133, 135.