Common Discrete Distributions.

1. Geometric Distribution.

- *Geometric Random Variable.*
  - **a.** Consider a Bernoulli trial with success probability p.
  - **b.** Perform this Bernoulli trial until a success is observed.
  - **c.** Let X represent the number of trials needed to get the first success.
- *Geometric Probability Distribution.* $X \sim g(p)$

$$p(x) = p(1-p)^{x-1} \qquad x = 1, 2, \ldots$$

  and 0 elsewhere.
- *Properties.*
  - **a.** $E(X) = \frac{1}{p}$.
  - **b.** $Var(X) = \frac{(1-p)}{p^2}$.

2. Binomial Distribution.

- *Binomial Random Variable.*
  - **a.** Consider a Bernoulli trial with success probability p.
  - **b.** Perform this Bernoulli trial n times.
  - **c.** Let X represent the number of successes out of the n trials.
- *Binomial Probability Distribution.* $X \sim b(x; n, p)$

$$p(x) = \binom{n}{x} p^x (1-p)^{n-x} \qquad x = 0, 1, \ldots, n$$

  and 0 elsewhere.
- *Properties.*
  - **a.** $E(X) = np$.
  - **b.** $Var(X) = np(1-p)$.

3. Hypergeometric Distribution.

- *Hypergeometric Random Variable.* Suppose a sample of n objects is randomly selected (without replacement) from a population of N objects containing M successes. If X denote the number of successes out of the n selected objects, then X follows the *hypergeometric distribution.*
- *Hypergeometric Probability Distribution.* $X \sim h(x; n, M, N)$

$$p(x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$$

  for integer x satisfying $\max(0, n-N+M) \leq x \leq \min(n, M)$.
- *Properties.*
  - **a.** $E(X) = n\left(\frac{M}{N}\right)$.
  - **b.** $Var(X) = \left(\frac{N-n}{N-1}\right) n \left(\frac{M}{N}\right)\left(1 - \frac{M}{N}\right)$.

4. Negative Binomial Distribution.

- *Negative Binomial Random Variable.*
  - **a.** Consider a Bernoulli trial with success probability p.
  - **b.** Perform this Bernoulli trial until a total of $r > 0$ successes have been observed.
  - **c.** Let X represent the number of failures that precede the rth success.
  - **d.** Let Y represent the number of trials needed to get the r successes.
- *Negative Binomial Probability Distribution.* $X \sim nb(x; r, p)$ and $Y \sim nb(y; r, p)$

$$p(x) = \binom{x+r-1}{r-1} p^r (1-p)^x \qquad x = 0, 1, 2, \ldots$$

  and 0 elsewhere, and

$$p(y) = \binom{y-1}{r-1} p^r (1-p)^{y-r} \qquad y = r, r+1, r+2, \ldots$$

  and 0 elsewhere.
- *Properties.*
  - **a.** $E(X) = \dfrac{r(1-p)}{p}; E(Y) = \dfrac{r}{p}.$
  - **b.** $Var(X) = \dfrac{r(1-p)}{p^2}; Var(Y) = \dfrac{r(1-p)}{p^2}.$

5. Poisson Probability Distribution. A random variable X is said to have a *Poisson distribution* with parameter $\lambda (\lambda > 0)$ if the pmf of X is

$$p(x; \lambda) = \frac{e^{-\lambda} \lambda^x}{x!} \qquad x = 0, 1, 2, \ldots$$

- *Properties.*
  - **a.** $E(X) = \lambda$.
  - **b.** $Var(X) = \lambda$.
- *Binomial Approximation.* Suppose that in the binomial pmf $b(x; n, p)$, we let $n \to \infty$ and $p \to 0$ in such a way that np approaches a value $\lambda > 0$. Then $b(x; n, p) \to p(x; \lambda)$.

6. Recommended problems:
Section 3.4: (pp. 110-114) # 37, 39, 45, 51, 53, 55, 61, 65.
Section 3.5: (pp. 119-121) # 67, 71, 73, 77, 81, 85.
Section 3.6: (pp. 123-125) # 91, 93, 97, 101.
Section 3.7: (pp. 128-130) # 105, 107, 109, 111, 115, 117.
Section 3.8: (pp. 136-138) # 121, 123, 125, 127, 131, 135, 137, 139.