Moments and Moment-Generating Functions.

- **Definitioin 3.12.** The kth *moment of a random variable* Y taken <u>about the origin</u> is defined to be $E(Y^k)$ and is denoted by μ'_k.
- **Definitioin 3.13.** The kth *moment of a random variable* Y taken <u>about its mean</u>, or the kth *central moment of* Y, is defined to be $E[(Y-\mu)^k]$ and is denoted by μ_k.
- **Definitioin 3.14.** The *moment-generating function* $m(t)$ for a random variable Y is defined to be $m(t) = E(e^{tY})$. We say that a moment-generating function for Y exists if there exists a positive constant b such that $m(t)$ is finite for $|t| \le b$.

  Example. Find the *moment-generating function* $m(t)$ for a Poisson distributed random variable with mean λ.

  Solution:

- **Theorem 3.12.** If $m(t)$ exists, then for any positive integer k,

$$\left.\frac{d^k m(t)}{dt^k}\right|_{t=0} = m^{(k)}(0) = \mu'_k.$$

  In other words, if you find the kth derivative of $m(t)$ with respect to t and then set $t = 0$, the result will be μ'_k.

  Proof:

- **Recommended problems:**
  Section 3.9: (pp. 142-143) # 145, 146, 147, 148, 153, 158, 159.

Section 3.11: Tschebysheff's Theorem

- **Theorem 3.14. Tchebysheff's Theorem:** Let Y be a random variable with mean μ and finite variance σ^2. Then, for any constant $K > 0$

$$P(|Y - \mu| < k\sigma) \geq 1 - \frac{1}{k^2} \text{ or } P(|Y - \mu| \geq k\sigma) \leq \frac{1}{k^2}.$$

Example 3.28 (page 147): The number of customers per day at a sales counter, Y, has been observed for a long period of time and found to have mean 20 and standard deviation 2. The probability distribution of Y is not known. What can be said about the probability that, tomorrow, Y will be greater than 16 but less than 24?

- **Recommended problems:**
Section 3.11: (pp. 147-149) # 167, 169, 173, 177.