Common Continuous Distributions

- **Uniform Distribution.** If $\theta_1 < \theta_2$, a random variable X is said to have a continuous *uniform probability distribution* on the interval $(\theta_1.\theta_2)$ if and only if the density function of X is

$$f(x) = \frac{1}{\theta_2 - \theta_1}, \qquad \theta_1 \le x \le \theta_2$$

  and 0 elsewhere.

  Remarks:

  1. $E(X) = \frac{1}{2}(\theta_1 + \theta_2)$.

  2. $Var(X) = \frac{1}{12}(\theta_2 - \theta_1)^2$.

  3. $F(x) = \dfrac{x - \theta_1}{\theta_2 - \theta_1}, \qquad x \ge 0.$

- **Normal Distribution.** A continuous r.v. X is said to have a *normal distribution* with parameters μ and σ (or μ and σ^2), where $-\infty < \mu < \infty$ and $\sigma > 0$, if the pdf of X is

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2\sigma^2}(x-\mu)^2} \qquad -\infty < x < \infty$$

  Remarks:

  1. $E(X) = \mu$.

  2. $Var(X) = \sigma^2$.

- **Gamma Distribution.** A continuous random variable X is said to have a *gamma distribution* with parameters α and β, where $\alpha > 0, \beta > 0$, if the pdf of X is

$$f(x) = \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta} \qquad x \ge 0$$

  where, the *gamma function* $\Gamma(\alpha)$ is defined by

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x}\, dx \qquad \alpha > 0.$$

  Remarks:

  1. $E(X) = \alpha\beta$.

  2. $Var(X) = \alpha\beta^2$.

- **Special Cases of the Gamma Distribution:**

  1. *Standard Gamma Distribution.* When $\beta = 1$.
  A continuous random variable X is said to have a *standard gamma distribution* if the pdf of X is

$$f(x) = \frac{1}{\Gamma(\alpha)} x^{\alpha-1} e^{-x} \qquad x \ge 0$$

  Find: (a) $E(X) =$ (b) $Var(X) =$

  2. *Exponential Distribution.* When $\alpha = 1$ and $\beta = \frac{1}{\lambda}$.
  A continuous random variable X is said to have an *exponential distribution* with parameter $\alpha > 0$ if the pdf of X is

$$f(x) = \lambda e^{-\lambda x} \qquad x \ge 0$$

  Find: (a) $E(X) =$ (b) $Var(X) =$

3. *Chi-Squared Distribution.* When $\alpha = \nu/2$ and $\beta = 2$.

A continuous random variable X is said to have a *chi-squared distribution* with parameter $\nu > 0$ if the pdf of X is

$$f(x) = \frac{1}{2^{\nu/2}\Gamma(\nu/2)} x^{\nu/2-1} e^{-x/2} \qquad x \geq 0$$

The parameter ν is called the *number of degrees of freedom* (df) of X. The symbol χ^2 is often used in place of "chi-squared".

Find: (a) $E(X) =$ (b) $Var(X) =$

- **Weibull Distribution.** A random variable X is said to have a *Weibull distribution* with parameters α and β $(\alpha > 0, \beta > 0)$ if the pdf of X is

$$f(x) = \frac{\alpha}{\beta^\alpha} x^{\alpha-1} e^{-(x/\beta)^\alpha} \qquad x \geq 0$$

Remarks:

1. $E(X) = \beta\Gamma\left(1 + \frac{1}{\alpha}\right)$.

2. $Var(X) = \beta^2 \left\{ \Gamma\left(1 + \frac{2}{\alpha}\right) - \left[\Gamma\left(1 + \frac{1}{\alpha}\right)\right]^2 \right\}$.

3. $F(x) = 1 - e^{-(x/\beta)^\alpha} \qquad x \geq 0$

4. When $\alpha = 1$, this distribution reduces to the exponential distribution with parameter $\lambda = \frac{1}{\beta}$.

- **Lognormal Distribution.** A nonnegative r.v. X is said to have a *lognormal distribution* if the r.v. $Y = \ln(X)$ has a normal distribution. The resulting pdf of a lognormal r.v. when $\ln(X)$ is normally distributed with parameters μ and σ is

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma x} e^{-\frac{1}{2\sigma^2}[\ln(x)-\mu]^2} \qquad x \geq 0$$

Remarks:

1. $E(X) = e^{\mu+\sigma^2/2}$.

2. $Var(X) = e^{2\mu+\sigma^2}(e^{\sigma^2} - 1)$

3. $F(x) = P(X \leq x) = \Phi\left(\frac{\ln(x)-\mu}{\sigma}\right)$

- **Beta Distribution.** A random variable X is said to have a *beta distribution* with parameters α, β $(\alpha > 0, \beta > 0)$, A, and B if the pdf of X is

$$f(x) = \frac{1}{B-A} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \left(\frac{x-A}{B-A}\right)^{\alpha-1} \left(\frac{B-x}{B-A}\right)^{\beta-1} \qquad A \leq x \leq B$$

Remarks:

1. $E(X) = A + (B-A) \cdot \frac{\alpha}{\alpha+\beta}$

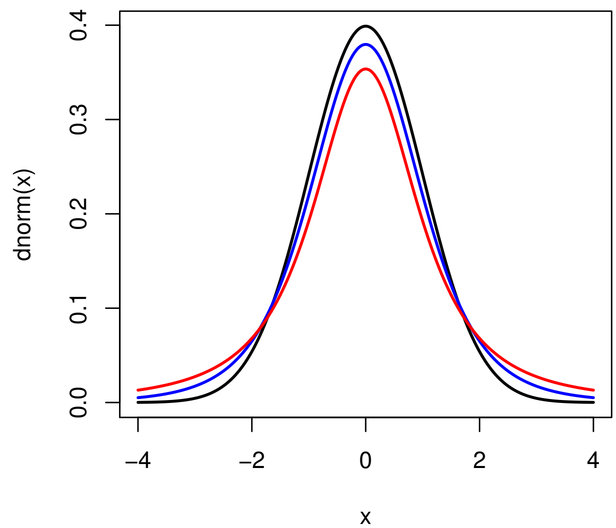
2. $Var(X) = \frac{(B-A)^2\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

3. When $A = 0, B = 1$, this distribution is called the *Standard Beta Distribution.*

$$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1} \qquad 0 \leq x \leq 1$$

- **Student $t-$distribution.** If $V \sim \chi^2(\nu)$ and $Z \sim N(0,1)$, then $X = \dfrac{Z}{\sqrt{V/\nu}}$ follows the $t-$distribution with ν degrees of freedom. A random variable X is said to have a *t-distribution* with parameter ν if the pdf of X is

$$f(x) = \frac{\Gamma((\nu+1)/2)}{\sqrt{\nu\pi}\,\Gamma(\nu/2)}(1+x^2/\nu)^{-(\nu+1)/2} \qquad -\infty < x < \infty$$



- **$F-$distribution.** If $V_1 \sim \chi^2(\nu_1)$ and $V_2 \sim \chi^2(\nu_2)$, then $X = \dfrac{V_1/\nu_1}{V_2/\nu_2}$ follows the $F-$distribution with parameters ν_1 and ν_2. A random variable X is said to have an *F-distribution* with parameters ν_1 and ν_2 if the pdf of X is

$$f(x) = \frac{\Gamma\left(\frac{\nu_1+\nu_2}{2}\right)\nu_1^{\frac{1}{2}\nu_1}\nu_2^{\frac{1}{2}\nu_2}}{\Gamma\left(\frac{\nu_1}{2}\right)\Gamma\left(\frac{\nu_2}{2}\right)}\frac{x^{\frac{1}{2}\nu_1-1}}{(\nu_2+\nu_1 x)^{\frac{1}{2}(\nu_1+\nu_2)}} \qquad x \geq 0$$