Moments and Moment-Generating Functions.

- **Definitioin 4.13.** Let Y is a continuous random variable.
  1. The kth *moment about the origin* is given by
     $\mu'_k = E(Y^k)$, for $k = 1, 2, \ldots$.
  2. The kth *moment about the mean*, or the kth *central moment*, is given by
     $\mu_k = E[(Y - \mu)^k]$ for $k = 1, 2, \ldots$.

- **Definitioin 4.14.** If Y is a continuous random variable, then the *moment-generating function* of Y is given by
  $$m(t) = E(e^{tY}).$$
  The moment-generating function is said to exist if there is a constant b such that $m(t)$ is finite for $|t| \le b$.

  Example 1. Find the *moment-generating function* $m(t)$ for a Gamma distributed random variable with parameters α and β.

  Solution:

- **Theorem 3.12.** If $m(t)$ exists, then for any positive integer k,
  $$\left.\frac{d^k m(t)}{dt^k}\right|_{t=0} = m^{(k)}(0) = \mu'_k.$$

  Example 2. If $Y \sim Gamma(\alpha, \beta)$, derive the mean and variance of Y using its moment-generating function.

- **Recommended problems:**
  Section 4.9: (pp. 206-207) # 136, 137, 139, 140, 145.

Section 4.10: Tschebysheff's Theorem

- **Theorem 4.13. Tchebysheff's Theorem:** Let Y be a random variable with mean μ and finite variance σ^2. Then, for any constant $K > 0$

$$P(|Y - \mu| < k\sigma) \geq 1 - \frac{1}{k^2} \text{ or } P(|Y - \mu| \geq k\sigma) \leq \frac{1}{k^2}.$$

Example 4.17 (page 208): Suppose that experience has shown that the length of time Y (in minutes) required to conduct a periodic maintenance check on a dictating machine follows a gamma distribution with $\alpha = 3.1$ and $\beta = 2$. Show that the probability that Y will be within 2 standard deviation of the mean time required to conduct a periodic maintenance check is at least 0.75.

- **Recommended problems:**
  Section 4.10: (pp. 209-210) # 147, 149, 153.