

Instructions: *Include all relevant work to get full credit.*

Homework 10

1. Suppose X has an exponential distribution with parameter $\beta = \frac{1}{\lambda}$,

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0, \\ 0 & \text{elsewhere.} \end{cases}$$

- a. Find cumulative distribution function of X , $F(x)$.
 - b. If $P(X > 2) = 0.0821$, find λ , $E(Y)$, and $P(X \leq 1.7)$.
2. Suppose X has an exponential distribution with parameter $\beta = \frac{1}{\lambda}$, show that $P(X > a + b | X > a) = P(X > b)$ for any $a > 0$ and $b > 0$.
3. Suppose $Y \sim \text{Gamma}(\alpha, \beta)$.
- a. Prove that $V(Y) = \alpha\beta^2$. *You may use the fact that $E(Y) = \alpha\beta$.*
 - b. If a is any positive or negative value such that $\alpha + a > 0$, show that $E(Y^a) = \frac{\beta^a \Gamma(\alpha + a)}{\Gamma(\alpha)}$.
 - c. Why did your answer in part (b) require that $\alpha + a > 0$?
 - d. Use the result in part (b) to give an expression for $E(1/Y)$.