STAT 441 - Mathematical Statistics I
Due: Friday (at the start of class), Oct. 13, 2017

Instructions: *Include all relevant work to get full credit.*

Homework 11

1. Suppose that the waiting time for the first customer to enter a retail shop after 9:00 AM is a random variable X has an exponential distribution with parameter $\beta = \frac{1}{\lambda}$,

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0, \\ 0 & \text{elsewhere.} \end{cases}$$

a. Find the moment-generating function for X.

b. Use the answer from part (a) to find $E(X)$ and $V(X)$.

2. If X is a random variable with moment-generating function $m_X(t)$ and Y is given by $Y = aX + b$, show that the moment-generating function of Y is $m_Y(t) = e^{tb} m_X(at)$.

3. If $Z \sim N(0,1)$, show that the moment-generating function of Z is $m_Z(t) = e^{\frac{1}{2}t^2} = exp(\frac{1}{2}t^2)$.

[Hint: Use the fact that the total area under the density function of $N(\mu = t, \sigma = 1)$ is 1.]

4. If $Z \sim N(0,1)$ and $Y = \sigma Z + \mu$, Y will be normally distributed with mean μ and standard deviation σ. Use the results from parts (2) and (3) to show that the moment-generating function of a normal random variable Y with mean μ and standard deviation σ is $m_Y(t) = exp\left(\mu t + \dfrac{t^2\sigma^2}{2}\right)$

5. Use the uniqueness of moment-generating functions to determine the exact distribution (give the name and parameter values) of the random variables that have each of the following moment-generating functions:

a. $m(t) = (1 - 4t)^{-2}$

b. $m(t) = 1/(1 - 3.2t)$

c. $m(t) = e^{-5t+6t^2}$