STAT 441 - Mathematical Statistics I
Due: Wednesday (at the start of class), Nov. 22, 2017

Instructions: *Include all relevant work to get full credit.*

Homework 19

1. Use the statistical package R to do all parts of problem 7.32 on page 368. *Include the R commands that you used.*
2. Do all parts of problem 7.37 on page 369.
3. Do all parts of problem 7.38 on page 369.
4. Do all parts of problem 7.39 on page 369.

Solutions:

1) (a) $t_{.05} = qt(.95, df=5) = 2.015048$

(b) $P(T^2 > (2.015048)^2) = P(|T| > 2.015048) = 1 - P(|T| < 2.015048)$
$= 1 - P(-2.015 < T < 2.015)$
$= 1 - [\,pt(2.015, df=5) - pt(-2.015, df=5)]$
$= 1 - (.95 - .05) = .10$

(c) $F_{.10} = qf(.90, df1=1, df2=5) = 4.06042$

(d) $t^2_{.05} = (2.015048)^2 = 4.06042 = F_{.10}$

(e) Since $T^2_{df=5} \sim f\text{-dist}(df_1=1, df_2=5) \Rightarrow P(T^2 > t^2_{.05}) = P(F > t^2_{.05}) = .10$
$\Rightarrow f_{.10} = t^2_{.05}$

---

2) (a) $W \sim \chi^2_{df=5}$, since $Y_i^2 \sim \chi^2_{df=1}$ by Thm 7.2

(b) $U = \sum_1^5 (Y_i - \bar{Y})^2 = \frac{(5-1)S^2}{\sigma^2} \sim \chi^2_{df=4}$ since $\sigma^2 = 1$ & $S^2 = \frac{\sum (Y_i - \bar{Y})^2}{5-1}$

by Theorem 7.3

(c) Since $Y_6^2 \sim \chi^2_{df=1}$ & $\sum_{i=1}^5 (Y_i - \bar{Y})^2 \sim \chi^2_{df=4} \Rightarrow \sum_{i=1}^5 (Y_i - \bar{Y})^2 + Y_6^2 \sim \chi^2_{df=5}$

---

3) (a) $\sqrt{5}\,Y_6/\sqrt{W} = \frac{Y_6}{\sqrt{W/5}} = \frac{Z}{\sqrt{W/\nu}} \sim t_{df=5}$

(b) $2Y_6/\sqrt{U} = \frac{Y_6}{\sqrt{U/4}} = \frac{Z}{\sqrt{W/\nu}} \sim t_{df=4}$

(c) $2(5\bar{Y}^2 + Y_6^2)/U = \frac{\frac{5\bar{Y}^2 + Y_6^2}{2}}{U/4}$
$= \frac{W_1/\nu_1}{W_2/\nu_2} \sim f_{df_1=2, df_2=4}$
Since $5\bar{Y}^2 \sim \chi^2_1$; $Y_6^2 \sim \chi^2_1$ and $U \sim \chi^2_{df=4}$

---

4) (a) By Theorem 7.1, $\bar{X}_i \sim N(\mu_i, \text{variance} = \sigma^2/n_i)$
So by Theorem 6.3, $\hat{\theta} \sim N(\text{mean} = \theta, \text{variance} = \sigma^2 \sum_1^k \frac{c_i^2}{n_i})$

(b) $\frac{SSE}{\sigma^2} = \frac{\sum_1^k (n_i - 1) S_i^2}{\sigma^2} = \sum_1^k \frac{(n_i - 1) S_i^2}{\sigma^2} \sim \chi^2_{df = \sum_1^k (n_i - 1)}$, since $\frac{(n_i - 1) S_i^2}{\sigma^2} \sim \chi^2_{df = n_i - 1}$

(c) $\frac{\hat{\theta} - \theta}{\sqrt{(\sum \frac{c_i^2}{n_i}) MSE}} = \frac{(\hat{\theta} - \theta)/\sigma\sqrt{\sum \frac{c_i^2}{n_i}}}{\sqrt{\frac{SSE}{\sigma^2} / \sum_1^k (n_i - 1)}} = \frac{Z}{\sqrt{W/\nu}} \sim t_{df = \nu = \sum_1^k (n_i - 1) = n - k}$
where $n = \sum_1^k n_i$