STAT 441 - Mathematical Statistics I
Due: Wednesday (at the start of class), Dec. 6, 2017

Instructions: *Include all relevant work to get full credit.*

Homework 21

1. Let $Y_1, Y_2, \ldots, Y_n$ be a random sample taken from a normal population with mean μ and variance σ^2. If $\bar{Y}$ and S^2 represent the sample mean and sample variance, respectively, we have shown in class that the quantity $T = \dfrac{\bar{Y} - \mu}{S/\sqrt{n}}$ has a $t-$distribution with $(n-1)$ degrees of freedom.

a. Show that the $(1-\alpha)100\%$ confidence interval for μ is given by $\bar{Y} \pm t_{\alpha/2}\left(\dfrac{S}{\sqrt{n}}\right)$, where $t_{\alpha/2}$ is the value of T such that $P(T > t_{\alpha/2}) = \alpha/2$.

b. Do problem 8.80 on page 430. Provide an interpretation of the confidence interval you obtained in the context of the problem.

2. Given two independent random samples: $X_1, X_2, \ldots, X_{n_1}$ from $N(\mu_1, \sigma^2)$ and $Y_1, Y_2, \ldots, Y_{n_2}$ from $N(\mu_2, \sigma^2)$. Let $\bar{X}$ and S_1^2 represent the sample mean and sample variance, respectively, of sample 1, and $\bar{Y}$ and S_2^2 represent the sample mean and sample variance, respectively, of sample 2.

a. Use definition 7.2 to show that $T = \dfrac{(\bar{X} - \bar{Y}) - (\mu_1 - \mu_2)}{\sqrt{\frac{S_p^2}{n_1} + \frac{S_p^2}{n_2}}} = \dfrac{(\bar{X} - \bar{Y}) - (\mu_1 - \mu_2)}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$ follows the $t-$distribution with $(n_1 + n_2 - 2)$ degrees of freedom, where $S_p^2 = \dfrac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$. [Hint: Use Theorem 7.2].

b. Use the result of part (a) to show that the $(1-\alpha)100\%$ confidence interval for $\mu_1 - \mu_2$ is given by

$$(\bar{X} - \bar{Y}) \pm t_{\alpha/2} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}},$$

where $t_{\alpha/2}$ is the value of T such that $P(T > t_{\alpha/2}) = \alpha/2$.

c. Use the result from part (b) to do all parts of problem 8.90 on page 433.