STAT 441 - Mathematical Statistics I
Due: Wednesday (at the start of class), Sept. 13, 2017

Instructions: *Include all relevant work to get full credit.*

Homework 2

1. A sample space consists of only 5 different outcomes (or simple events), E_1, E_2, E_3, E_4, and E_5. If $P(E_1) = P(E_2) = 0.12, P(E_3) = .37$, and $P(E_4) = 2P(E_5)$, find the probability of E_4 and E_5.

Let $P(X_5) = x \Rightarrow P(E_1)+P(E_2)+P(E_3)+P(E_4)+P(E_5) = 1$

$\Rightarrow .12 + .12 + .37 + 2x + x = 1 \Rightarrow 3x = 1 - .61 = .39$

$\Rightarrow x = .13$

$\Rightarrow P(X_5) = .13$ and $P(X_4) = .26$

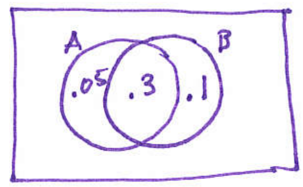
2. Suppose $P(A) = 0.35, P(B) = .4$, and $P(A \cap B) = .3$.

   a. Find $P(A^c)$.

   $= 1 - P(A) = 1 - .35 = .65$

   b. Find $P(A \cup B)$.

   $= P(A) + P(B) - P(A \cap B)$
   $= .35 + .4 - .3 = .45$



   c. Find $P(A|B)$.

   $$= \frac{P(A \cap B)}{P(B)} = \frac{.3}{.4} = .75$$

   d. Find $P(A^c|B)$.

   $$= \frac{P(A^c \cap B)}{P(B)} = \frac{.1}{.4} = .25$$

   e. Find $P(A^c|B^c)$.

   $$= \frac{P(A^c \cap B^c)}{P(B^c)} = \frac{P((A \cup B)^c)}{1 - P(B)} = \frac{1 - P(A \cup B)}{1 - P(B)} = \frac{1 - .45}{1 - .4} = \frac{.55}{.60} = \frac{11}{12}$$

   f. Are events A and B mutually exclusive? Explain why.

   No, because $P(A \cap B) = .3 \neq 0 \Rightarrow A \cap B \neq \emptyset$.

   g. Are events A and B independent? Justify your answer.

   No, because $P(A|B) = .75 \neq P(A) = .35$

3. Prove that $P(A|B) + P(A^c|B) = 1$.

Proof:

$$P(A|B) + P(A^c|B) = \frac{P(A \cap B)}{P(B)} + \frac{P(A^c \cap B)}{P(B)}$$

$$= \frac{P(A \cap B) + P(A^c \cap B)}{P(B)},$$

Note: $(A \cap B) \cup (A^c \cap B) \equiv B$ and are mutually exclusive

$$= \frac{P((A \cap B) \cup (A^c \cap B))}{P(B)} = \frac{P(B)}{P(B)} = 1. \quad \blacksquare$$