

Instructions: Include all relevant work to get full credit.

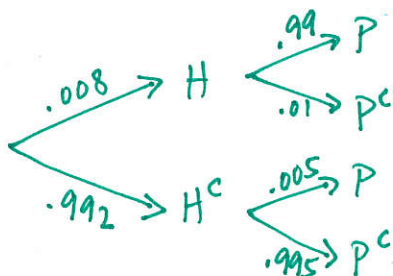
Homework 3

1. A communications network has a built-in safeguard system against failures. In this system if line I fails, it is bypassed and line II is used. If line II fails, it is bypassed and line III is used. The probability of failure of any one of these three lines is 0.01, and the failures of these lines are independent events. What is the probability that this system of three lines does not completely fail? [2 pts]

$$= 1 - (.01)^3 = .999999$$

2. In North America, the probability of a person having HIV is .008. A test for HIV yields either a positive or negative result. If a person has HIV, the probability of a positive test result is 0.99. If the person does not have HIV, the probability of a positive test result is 0.005. Let H be the event that a randomly selected person from North America has HIV and P be the event that the person is tested positive for HIV. Suppose one person is randomly selected from North America. [5 pts]

- a. Construct a tree diagram, showing the different possibilities (using H and P) and their corresponding probabilities.



- b. What is the probability that the person will test positive? Use 5 decimal places.

$$\begin{aligned} P_r(P) &= (.008)(.99) + (.992)(.005) \\ &= .00792 + .00496 = .01288 \end{aligned}$$

- c. What is the probability that the person doesn't have HIV given that his/her test came out to be positive? Use 3 decimal places.

$$P_r(H^c|P) = \frac{P(H^c \cap P)}{P(P)} = \frac{.00496}{.01288} \approx .385$$

3. If events A and B are independent, prove that A^c and B^c are also independent. [3 pts]

Proof:

$$P(A^c \cap B^c) \overset{\text{by De Morgan's Law}}{=} P((A \cup B)^c) \overset{\text{by Thm 2.7}}{=} 1 - P(A \cup B)$$

$$= 1 - (P(A) + P(B) - P(A \cap B)) \quad \text{by Thm 2.6}$$

$$= 1 - P(A) - P(B) + P(A)P(B) \quad \text{since } A \text{ and } B \text{ are independent}$$

$$= (1 - P(A))(1 - P(B))$$

$$= P(A^c)P(B^c)$$

Hence, A^c and B^c are independent.