STAT 441 - Mathematical Statistics I
Due: Wednesday (at the start of class), Sept. 20, 2017

Instructions: *Include all relevant work to get full credit.*

Homework 4

1. Let X be a random variable with $p(x)$ given in the table below.

x	1	2	3	4
$p(x)$	0.4	0.3	0.2	0.1

a. Find $E(X), E(1/X), E[X(X-1)]$, and $V(X)$. [2]

$E(x) = 1(.4) + 2(.3) + 3(.2) + 4(.1) = 2$

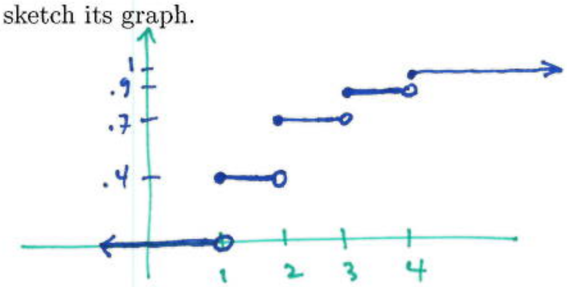
$E(\frac{1}{x}) = 1(.4) + \frac{1}{2}(.3) + \frac{1}{3}(.2) + \frac{1}{4}(.1) \approx .6417$

$E(x(x-1)) = 0(.4) + 2(.3) + 6(.2) + 12(.1) = 3$

$V(x) = (1-2)^2(.4) + (2-2)^2(.3) + (3-2)^2(.2) + (4-2)^2(.1) = 1$

b. Write the cumulative distribution function $F(x)$ and then sketch its graph. [2]

$$F(x) = \begin{cases} 0 & , x < 1 \\ .4 & , 1 \le x < 2 \\ .7 & , 2 \le x < 3 \\ .9 & , 3 \le x < 4 \\ 1 & , x \geq 4 \end{cases}$$



2. An oil prospector will drill a succession of holes in a given area to find a productive well. The probabiblity that he is successful on a given drill in 0.2. Assume that each drill is independent of one another.

a. If X denotes the number of holes drilled until the first productive well is found, what is the distribution of X? What is the probability that the third hole drilled is the first to yield a productive well? [1]

$X \sim \text{Geometric}(p = .2)$, $P(x=3) = (.2)(.8)^2 = 0.128$

b. What is the expected number of holes needed to be drilled to get the first hole to yield a productive well? [1]

$E(x) = \frac{1}{p} \Rightarrow E(x) = \frac{1}{.2} = 5$

3. Let Y denote a geometric random variable with probability of success p, If $Y \sim Geo(p)$

a. Show that for a positive integer a, $P(Y > a) = (1-p)^a$. [2]

$\Rightarrow P(Y=y) = p(1-p)^{y-1}, \; y = 1, 2, \ldots$

$\Rightarrow P(Y > a) = \sum_{y=a+1}^{\infty} P(Y=y) = p(1-p)^a + p(1-p)^{a+1} + \ldots$

$\left(\frac{a_1}{1-r}\right) = \frac{p(1-p)^a}{1-(1-p)} = \frac{p(1-p)^a}{p} = (1-p)^a$ ∎

b. Show that for positive integers a and b, $P(Y > a + b | Y > a) = P(Y > b) = (1-p)^b$ This is known as the *memoryless* property of the geometric distribution. [2]

$\Rightarrow P(Y > a+b \mid Y > a) = \frac{P(Y > a+b \cap Y > a)}{P(Y>a)} = \frac{P(Y > a+b)}{P(Y>a)}$

from part (a) $= \frac{(1-p)^{a+b}}{(1-p)^a} = (1-p)^b$ ∎

$= P(Y > b)$