STAT 441 - Mathematical Statistics I
Due: Friday (at the start of class), Sept. 22, 2017

Instructions: *Include all relevant work to get full credit.*

Homework 5

1. A particular concentration of a chemical found in polluted water has been found to be lethal to 40% of the fish that are exposed to the concentration for 24 hours. Twenty five (25) fish are placed in a tank containing this concentration of chemical in water for 24 hours. [3]

a. Let X denote the number of fish that survive out of the 25 fish placed in the tank. Give the name and the parameter values of the distribution of X.

b. Find the probability that exactly 14 survive. *Use 4 decimal places.*

c. Find the probability that at most 14 survive. *Use 4 decimal places.*

d. Find the probability that at least 12 survive. *Use 4 decimal places.*

e. Find the mean and variance of the number that survive.

2. Suppose there are n trials in a binomial experiment and we observe y_0 "successes," show that $P(Y = y_0)$ is maximized when $p = y_0/n$. *Make sure to check that you get a maximum and not a minimum at this point.*
[Hint: The maximum of $P(Y = y_0)$ and $\ln(P(Y = y_0))$ occur at the same place]. [3]

3. The *maximum likelihood estimator* for p is $\hat{p} = Y/n$, where $Y \sim$ binomial(n, p). Derive $E(\hat{p})$ and $V(\hat{p})$. [2]

4. If X has a geometric distribution with success probability p,

a. show that $P(X = \text{an odd integer}) = \dfrac{p}{1-(1-p)^2}$. [1]

b. show that $E(\hat{p}) = \dfrac{-p\ln(p)}{1-p}$, where $\hat{p} = \frac{1}{X}$ is the *maximum likelihood estimator* for p.
[Hint: If $|r| < 1, \sum_{i=1}^{\infty} \dfrac{r^i}{i} = -\ln(1-r)$.] [1]