STAT 441 - Mathematical Statistics I
Due: Monday (at the start of class), Sept. 25, 2017

Instructions: *Include all relevant work to get full credit.*

Homework 6

1. **Poisson Probability Distribution.** A random variable X is said to have a *Poisson distribution* with parameter $\lambda(\lambda > 0)$ if the pmf of X is

$$p(x; \lambda) = \frac{e^{-\lambda}\lambda^x}{x!} \qquad x = 0, 1, 2, \ldots$$

   a. Prove that $\sum_{x=0}^{\infty} p(x; \lambda) = 1$. [2]

   b. Prove that $E(X) = \lambda$. [2]

   c. Prove that $Var(X) = \lambda$. [2]

2. **Negative Binomial Distribution.** Let Y denote the number of independent identical Bernoulli trials needed until the rth success is observed. If the probability of success in each trial is p, then $Y \sim NB(y; r, p)$. That is,

$$p(y) = \binom{y-1}{r-1} p^r (1-p)^{y-r} \qquad y = r, r+1, r+2, \ldots$$

   and 0 elsewhere.

   Prove that $Var(Y) = \dfrac{r(1-p)}{p^2}$. [4]