STAT 441 - Mathematical Statistics I
Due: Friday (at the start of class), Sept. 29, 2017

Instructions: *Include all relevant work to get full credit.*

Homework 7

1. Suppose X has a binomial distribution with n trials and probability of success p.

a. Show that the moment-generating function for X is

$$m(t) = (pe^t + q)^n, \qquad \text{where } q = 1 - p.$$

[Hint: Think of the binomial expansion $(a+b)^n = \sum_{i=0}^{n} \binom{n}{i} a^{n-i} b^i$]

b. Use the result from part (a) and Theorem 3.12 to show that the mean of X is np.

c. Use the results from parts (a), (b), and Theorem 3.12 to show that the variance of X is $np(1-p)$.

2. If Y has a geometric distribution with probability of success p, show that the moment-generating function for Y is

$$m(t) = \frac{pe^t}{1 - qe^t}, \qquad \text{where } q = 1 - p.$$

[Recall: The sum of an infinite geometric series is $\sum_{i=0}^{\infty} ar^i = \frac{a}{1-r}$ when $|r| < 1$.]

3. Use the uniqueness of moment-generating functions to determine the exact distribution (*give the name and parameter values*) of the random variables that have each of the following moment-generating functions:

a. $m(t) = e^{5(e^t - 1)}$

b. $m(t) = [(1/5)e^t + (4/5)]^{10}$

c. $m(t) = \dfrac{2e^t}{5 - 3e^t}$