

**Instructions:** *Include all relevant work to get full credit.*

### Homework 8

1. A gas station operates two pumps, each of which can pump up to 10,000 gallons of gas in a month. The total amount of gas pumped at the station in a month is a random variable  $Y$  (measured in 10,000 gallons) with a probability density function given by

$$f(y) = \begin{cases} y, & 0 \leq y < 1, \\ 2 - y, & 1 \leq y \leq 2, \\ 0 & \text{elsewhere.} \end{cases}$$

- a. Graph  $f(y)$ .
- b. Find  $F(y)$  and graph it.
- c. Find the probability that the station will pump between 8000 and 12,000 gallons in a particular month.
- d. Given that the station pumped more than 10,000 gallons in a particular month, find the probability that the station pumped more than 15,000 gallons during the month.
- e. Use definition 4.5,  $E(Y) = \int_{-\infty}^{\infty} yf(y) dy$ , to show that  $E(Y) = 1$ .  
[Hint: Split the integral into 2 parts – one for  $0 \leq y < 1$  and another for  $1 \leq y \leq 2$ .]
- f. Use the fact that  $V(Y) = E(Y^2) - (E(Y))^2$  to show that  $V(Y) = \frac{1}{6}$ .
- g. Calculate the probability that  $Y$  will be within 2 standard deviations from the mean. Does it satisfy the Tchebysh-eff's Theorem?