STAT 441 - MATHEMATICAL STATISTICS I
DUE: WEDNESDAY (AT THE START OF CLASS), OCT. 11, 2017

**Instructions:** *Include all relevant work to get full credit.*

## Homework 9

1. The normal resting heart rate for adults ranges from 60 to 100 beats a minute. But well-trained athlete have a much lower normal resting heart rate. Suppose the pulse rates of well-trained athletes are approximately normally distributed with a mean of 45 beats per minute (bpm) and a standard deviation of 6 bpm.

   **a.** What proportion of well-trained athletes has a pulse rate of

      i. at least 60 bpm? *Sketch an appropriate density curve and shade the area under the curve that corresponds to this question.*

      ii. between 50 bpm and 60 bpm?

   **b.** What is the minimum pulse rate of those well-trained athletes in the upper extreme 2.5%?

2. Wires manufactured for use in a computer system are specified to have resistances between 0.12 and 0.14 ohms. The actual measured resistances of the wires produced by company $A$ have a normal probability distribution with mean 0.13 ohm and standard deviation of 0.005 ohm.

   **a.** What is the probability that a randomly selected wire from company $A$'s production will meet the specification?

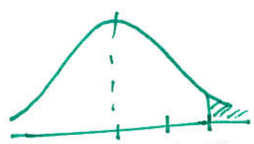
   **b.** If five of these wires are used in each computer system and all are selected from company $A$, what is the probability that at least one of the five wires in a randomly selected system will not meet the specifications?

3. If $Y \sim N(\mu, \sigma^2)$, show that the maximum value of the normal density is $1/(\sigma\sqrt{2\pi})$, and occurs when $y = \mu$. *Again, don't forget to check that you get a maximum and not a minimum at this critical point.*

4. Assume that $Y \sim N(\mu, \sigma^2)$. After observing a value of $Y$, a student constructs a rectangle with length $L = 3|Y|$ and width $W = 2|Y|$. Let $A$ denote the area of the rectangle. Determine the $E(A)$. [Hint: Note that $|Y|^2 = Y^2$]

Solutions:

1. (a) $X \sim N(\mu = 45, \sigma = 6)$




$P(X \geq 60)$

$= P\left(\frac{X-\mu}{\sigma} \geq \frac{60-45}{6}\right) = P(Z \geq 2.5) = \boxed{.0062}$

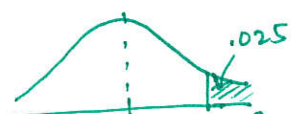
(b) $P(50 < X < 60)$

$= P\left(\frac{50-45}{6} < \frac{X-\mu}{\sigma} < \frac{60-45}{6}\right)$

$= P(.83 < Z < 2.5)$

$= \Phi(2.5) - \Phi(.83) = .9938 - .7967 = \boxed{.1972}$

(c) $P(X > ?) = .025$




$\Rightarrow P\left(Z > \frac{?-45}{6}\right) = .05$

$\Rightarrow \frac{?-45}{6} = 1.96 \Rightarrow ? = 45 + 1.96(6) = \boxed{56.76}$

2. a) $X \sim N(\mu = 0.13, \sigma = .005)$

$\Rightarrow P(.12 < X < .14) = P(-2 < Z < 2)$

$= \Phi(2) - \Phi(-2) = .9772 - .0228 = \boxed{.9544}$

b) $Y$ = no. of wires that do not meet specifications

$\Rightarrow Y \sim Bin(n=5, p=.0456)$

$P(Y \geq 1) = 1 - P(Y \leq 0)$

$= 1 - \binom{5}{0}(.0456)^0(.9544)^5$

$= 1 - .792 = \boxed{.208}$

3. $f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$

$\Rightarrow L(x) = \ln f(x) = \ln\left(\frac{1}{\sqrt{2\pi}\sigma}\right) - \frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2$

$\frac{dL}{dx} = -\left(\frac{x-\mu}{\sigma}\right)\left(\frac{1}{\sigma}\right) = 0 \Rightarrow x = \mu$

$\frac{d^2L}{dx^2} = -\frac{1}{\sigma^2} < 0 \Rightarrow L(x)$ and $f(x)$ is maximum at $x = \mu$. Therefore, $\max f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{\mu-\mu}{\sigma}\right)^2} = \frac{1}{\sqrt{2\pi}\sigma}$.

4. $A = L \times W = 3|Y| \times 2|Y| = 6|Y|^2 = 6Y^2$

$\Rightarrow E(A) = E(6Y^2) = 6E(Y^2)$

$= 6\left(V(Y) + (E(Y))^2\right)$

$= 6(\sigma^2 + \mu^2).$