Applied Economics, 2003, **35**, 875–880

A new criteria for selecting the optimum lags in Johansen's cointegration technique

MOHSEN BAHMANI-OSKOOEE* and TAGGERT J. BROOKS

Department of Economics and The Centre for Research on International Economics, The University of Wisconsin-Milwaukee, Milwaukee, WI 53201, USA and *Department of Economics, The University of Wisconsin-La Crosse, La Crosse, WI 54601, USA*

Several test statistics like Akaike Information Criterion (AIC) or Schwarz Bayesian Criterion (SBC) are used to select the order of Vector Autoregressive Models (VAR) in Johansen's cointegration technique, but not the appropriate cointegrating vector in case of multiple vectors. In this note goodness of fit is introduced as a criterion to select the lag length as well as the appropriate vector simultaneously.

I. INTRODUCTION

Since the introduction of cointegration technique by Engle and Granger (1987) and Johansen (1988) and Johansen-Juselius (1990) almost all economic relations have been reexamined. Johansen's technique is said to be more efficient and more powerful in that it not only allows for a feedback effect among the variables that enter into cointegrating space, but it is also based on the maximum likelihood procedure for estimating the long-run cointegrating vectors. Further more, when there are more than two variables in any reduced form model, it identifies the number of cointegrating vectors. Studies that have applied Johansen's technique are too numerous to be cited. However, there are two issues that have recently raised some concerns and questions about the technique. The first is that the results are sensitive to the choice of the order of the VAR in the procedure (see for example Toda, 1994; Bahmani-Oskooee, 1995). The second issue is concerned with multiple cointegrating vectors in case there is more than one. Usually, when there is more than one vector, they provide totally different coefficient estimates with different signs. In this case a researcher is confused about the choice of one vector over the other.

This paper introduces the goodness of fit not only as a criteria for selecting the optimum number of lags in the Johansen's cointegration technique, but also as a criterion for selecting the right vector in case of multiple cointegrating vectors. By estimating bilateral import and export demand models between the USA and the UK, it is demonstrated that when goodness of fit is used as a criterion for the choice of lag length and the cointegrating vector, the sign and size of the estimated coefficients are in line with theoretical expectations. To this end, the import and export demand models in Section II are introduced. The empirical results are reported in Section III. Section IV provides a summary. Finally, the appendix lists the definition and the sources of the data.

II. THE IMPORT AND EXPORT DEMAND MODELS

The traditional approach to assess the effectiveness of real devaluation or real depreciation on the trade flows of a country has been one of estimating the import and export demand elasticities. Following the small country assumption, the common models employed by previous researchers are the ones in which aggregate imports is related to a measure of domestic income and to relative prices. In the export demand function, the real quantity of exports is related to a measure of world income and again to relative prices. If the sum of import and export demand relative price elasticities add up to more than unity (i.e. the Marshall-Lerner condition is satisfied), real devaluation or depreciation is said to be effective in improving a coun-

* Corresponding author. E-mail: bahmani@uwm.edu

Applied Economics ISSN 0003–6846 print/ISSN 1466–4283 online © 2003 Taylor & Francis Ltd
http://www.tandf.co.uk/journals
DOI: 10.1080/00036840210129419

try's trade balance. All studies that have estimated the Marshall-Lerner condition, have employed data from developed or less developed countries, only at the aggregate level. Examples include Kreinin (1967, 1973), Houthakker and Magee (1969), Khan (1974), Warner and Kreinin (1983), Bahmani-Oskooee (1986, 1998), and Marquez (1990). The models are modified so that they conform to bilateral trade between two countries, say the USA and the UK, rather than one country versus the rest of the world. The modified models take the following forms:

$$\text{Ln} M_{u.s.t.} = \alpha + \beta \, \text{Ln} Y_{u.s.t.} + \gamma \, \text{Ln}\,(P_{u.s.} \cdot R/P_{u.k.})_t + \varepsilon \quad (1)$$

where $M_{u.s.}$ is the US imports from the UK; $Y_{u.s.}$ is the US real income; $P_{u.s.}$ is the US price level; $P_{u.k.}$ is the UK price level; and R is the bilateral nominal exchange rate defined as number of British pounds per US dollar. Based on this definition of exchange rate, a decline in real exchange rate will be an indication of real depreciation of the dollar. If this real depreciation is to lower the US imports from the UK, it would be expected that estimates of $\gamma > 0$. The income elasticity is also expected to be positive. The UK demand for the US exports is formulated as follows:

$$\text{Ln}\, X_{u.s.t.} = \alpha' + \beta' \text{Ln}\, Y_{u.k.t.} + \gamma' \text{Ln}(P_{u.s.} \cdot R/P_{u.k.})_t + \varepsilon'_t \quad (2)$$

where $X_{u.s.}$ is the US exports to the UK; $Y_{u.k.}$ is the UK real income. If real depreciation of the dollar is to stimulate America's exports to the UK, estimate of γ' must be negative. Again, the income elasticity (β') is expected to be positive.

Note that in the models, $P_{u.s.} \cdot R/P_{u.k.}$ is usually referred to as the real exchange rate. In the next section we try to provide an estimate of the long-run import and export demand models outlined by Equations 1 and 2.

III. EMPIRICAL RESULTS

Using quarterly data over 1973I–1996II period, an attempt is made to establish the long-run relationship among the variables of both the import and export demand models using the Johansen-Juselius (1990) cointegration technique. However, before applying the technique the degree of integration of each variable must be determined. To this end the KPSS test is employed. Kwiatkowski *et al.* (1992) have introduced this powerful test (known as KPSS test) in which the null hypothesis is stationarity of a variable versus an alternative of a unit root. The KPSS test assumes that a time series variable Z_t could be decomposed into the sum of a deterministic trend, a random walk, and a stationary error as in Equation 3 below:

$$Z_t = at + r_t + \varepsilon_t \quad (3)$$

where r_t is a random walk as in Equation 4 below:

$$r_t = r_{t-1} + u_t \quad (4)$$

The stationarity of Z_t is tested by simply testing whether $\sigma_u^2 = 0$. To this end, the residuals (call them e_t) from the regression of Z_t on a constant term and a trend term are used to form the following KPSS statistic:[1]

$$T^{-2} \Sigma S_t^2 / S^2(1) \quad (5)$$

where $S_t = \sum_{i=1}^{t} e_i$ and $s^2(1) = T^{-1} \sum_{t=1}^{T} e_t^2 + 2T^{-1} \sum_{s=1}^{1} w(s,1) \sum_{t=s+1}^{T} e_t e_{t-2}$. Following KPSS, the Bartlett window where $w(s,1) = 1 - s/1 + 1$ is used in our calculations. The results of this test for both level stationarity and trend stationarity for different values of truncation lag 1 are reported in Table 1.

It is clear from Panel A of Table 1 that the null of level stationarity is rejected, using 10% critical value, for all variables when truncation lag is less than or equal to five.

Table 1. *The KPSS test results*

	Lag truncation parameter								
Variable	0	1	2	3	4	5	6	7	8
Panel A: the KPSS statistics for null of level stationary. (The 5% and 10% critical values are 0.463 and 0.347 respectively.)									
Ln $M_{u.s.}$	2.801	2.135	1.728	1.454	1.256	1.108	0.992	0.898	0.820
Ln $Y_{u.s.}$	3.097	2.334	1.878	1.575	1.359	1.197	1.072	0.971	0.888
Ln *REX*	0.767	0.582	0.475	0.406	0.357	0.322	0.295	0.271	0.248
Ln $X_{u.s.}$	2.561	1.955	1.591	1.344	1.165	1.031	0.928	0.842	0.770
Ln $Y_{u.k.}$	3.059	2.304	1.852	1.552	1.338	1.178	1.054	0.954	0.871
Panel B: the KPSS statistics for null of trend stationary. (The 5% and 10% critical values are 0.146 and 0.119 respectively.)									
Ln $M_{u.s.}$	0.134	0.113	0.100	0.091	0.083	0.078	0.074	0.070	0.066
Ln $Y_{u.s.}$	0.110	0.084	0.069	0.059	0.053	0.048	0.045	0.041	0.038
Ln *REX*	0.137	0.104	0.086	0.073	0.065	0.059	0.054	0.050	0.046
Ln $X_{u.s.}$	0.154	0.124	0.106	0.093	0.083	0.076	0.070	0.065	0.060
Ln $Y_{u.k.}$	0.263	0.202	0.165	0.140	0.123	0.111	0.101	0.093	0.085

[1] Outlined by their equation (13) on page 165.

However, the null of trend stationarity is rejected for all variables when truncation lag is zero. Further analysis using the ADF test revealed that all variables are indeed non-stationary and they become stationary after being differenced once (including or excluding the trend term).

Assuming all variables are integrated of order one, the application of the Johansen-Juselius cointegration technique is applied to each of the models separately:

Analysis of import demand Equation 1

Let us first consider the US import demand Equation 1. Since there are three $I(1)$ variables in this equation, there must be a maximum of two cointegrating vectors. Regardless of whether the variables are cointegrated or not, it is estimated, normalized on Ln $M_{u.s.}$ by setting its coefficient at -1.00, and the two vectors are reported for each lag length beginning with one lag and ending at eight lags. The MFIT3.0 statistical package employed here allows maximum of eight lags. The results of this first step are reported in Table 2.

In the second step, the fitted values from each vector in each case against the actual values of $Ln\ M_{u.s.}$ are put in a graph. Whichever vector gives the closest fitted value against the actual value, not only determines the lag length, but also the specific vector within that lag structure. After trying all 16 fitted values from Table 2, the second vector of the case with three lags gave us the best fitted values.[2] Figure 1 plots the fitted values of both vectors against the actual values.

The next question needed to be addressed is whether the variables in the US import demand equation are cointegrated when the order of VAR is set at three. Are there really two cointegrating vectors, as reported in Table 2? To answer these two questions, the Johansen-Juselius procedure introduces two test statistics known as λ-max and trace tests which determine the number of cointegrating vectors, r, among the variables in the cointegrating space. These results are reported in Table 3.

As can be seen from Panel A of Table 3, the null of no cointegration is rejected by both λ-max and the trace tests due to the fact that both calculated statistics are larger than critical values (indicated by an * next to the statistic). The null of at most one cointegrating vector is also rejected in favour of $r = 2$. Thus, there are two cointegrating vectors. Although these vectors were reported in Table 2, they are reported again in Panel B of Table 3 with some additional information with regard to the significance of each estimated coefficient. For each coefficient estimate a χ^2 statistic to determine the significance of each variable is reported inside the bracket beneath the estimated coefficients. Johansen and Juselius (1990, p. 194) actually call this test the exclusion test which is based on the estimates of eigenvalues of unrestricted and restricted cointegrating space.[3] It is clear from the results that both income and real exchange rate do carry their expected positive signs and they are both highly significant, indicating that they do belong to the cointegrating space. Concentrating on the second vector which gave us the best fit, while the income elasticity is greater than unity, the relative price elasticity is less than one. Thus, the US imports from the UK seems to be price inelastic. How is our proposed criteria compared to others in the literature? Two other criteria were applied to select the lag length of the VAR. The Akaike Information Criteria (AIC = 496.2) identified two lags as optimal and the Schwarz Baysian Criterion (SBC = 478.4) identified

Table 2. *Coefficient estimates of two vectors for different order of VAR*

Lag length in the VAR	Ln $M_{u.s.}$	Ln $Y_{u.s.}$	Ln $(P_{u.s.} \cdot R/P_{u.k.})$	Constant	MSSR
1	−1.00	2.5325	0.9776	−13.68	0.0828
	−1.00	2.1005	0.1134	−11.52	0.0132
2	−1.00	2.4380	0.7984	−13.25	0.0364
	−1.00	2.0737	−0.1324	−11.55	0.0194
3	−1.00	3.3470	3.1106	−17.58	0.5029
	−1.00	2.2688	0.2621	−12.57	0.0116
4	−1.00	3.0470	2.2147	−16.30	0.2360
	−1.00	2.2999	0.1949	−12.86	0.0117
5	−1.00	3.7647	4.3537	−19.45	0.9074
	−1.00	2.2495	0.1609	−12.47	0.0134
6	−1.00	3.2996	2.9429	−17.45	0.0135
	−1.00	2.1382	−0.0038	−11.79	0.0148
7	−1.00	3.3409	3.1032	−17.58	0.4548
	−1.00	1.9793	−0.0728	−10.69	0.0160
8	−1.00	−35.0380	−71.3410	207.69	155.50

[2] This approach is equivalent to minimizing sum of squared residuals associated with each vector, as can be seen by MSSR (mean sum of squared residuals) statistic, also reported in Table 2.
[3] For a detailed explanation and application of the exclusion test see Bahmani-Oskooee (1996).

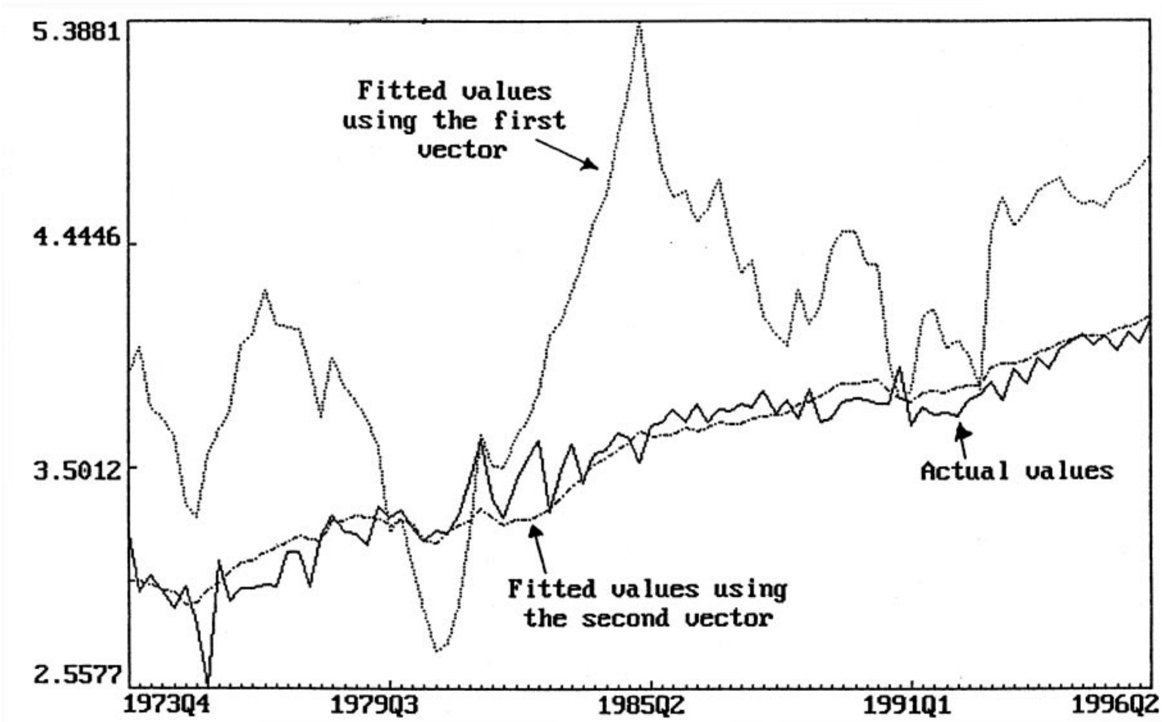

Fig. 1. *Fitted us actual values of both vectors when the order of VAR = 3*

Table 3. *Johansen's maximum likelihood results for the US import demand (r = number of cointegrating vectors; lags in the VAR = 3)*

Panel A: the results of λ-max and trace tests

Null	Alternative	λ-max statistic	90% critical value	Trace statistic	90% critical value
$r = 0$	$r = 1$	23.88*	19.76	42.29*	32.00
$r <= 1$	$r = 2$	14.37*	13.75	18.40*	17.85
$r <= 2$	$r = 3$	4.03	7.52	4.03	7.52

Panel B: estimates of cointegrating vectors

$\text{Ln}\, M_{u.s.}$	$\text{Ln}\, Y_{u.s.}$	$\text{Ln}\,(P_{u.s.} \cdot R/P_{u.k.})$	Constant
−1.00	3.35	3.11	−17.58
−1.00	2.27	0.26	−12.57
[10.6]	[11.1]	[12.4]	[10.9]

Note: In panel B the degrees of freedom of χ^2 statistic is the same as number of cointegrating vectors, i.e., 2. The critical value of $\chi^2_{(2)} = 5.99$, at the 5% level of significance.

only one lag. While these two later criteria can determine the order of VAR, they cannot identify the appropriate vector when there is more than one cointegrating vector. The proposed goodness of fit criteria achieves the two goals of selecting the order of VAR and the appropriate vector at the same time, thus, it should be preferred.

Indeed, Cheung and Lai (1993, p. 322) who investigated the performance of AIC and SIC, showed that they perform poorly in the presence of moving average dependence.

Analysis of the export demand Equation 2

The same method of selecting the lag length and the specific cointegrating vector for the US export demand model outlined by Equation 2 is followed. Just like the import demand model, the best fit was obtained by the second vector of the case when the order of VAR was set at 3. These results are reported in Table 4.

Following the same explanation as for the US import demand, panel A of Table 4 reveals that there are two cointegrating vectors using at least the λ-max statistic. Estimates of the vectors are reported in panel B which shows that all coefficient estimates are highly significant. Figure 2 depicts the fitted values using the coefficient estimates from first and second vectors against the actual $\text{Ln}X_{u.s.}$ variable.

As indicated above and could be observed from Figure 2, again the second vector provides the best fit. It is interest-

Table 4. *Johansen's maximum likelihood results for the US exports (r = number of cointegrating vectors; lags in the VAR = 3)*

Panel A: the results of λ-max and trace tests

Null	Alternative	λ-max statistic	90% critical value	Trace statistic	90% critical value
$r = 0$	$r = 1$	23.21*	19.76	36.71*	32.00
$r <= 1$	$r = 2$	14.40*	13.75	16.48*	17.85
$r <= 2$	$r = 3$	2.08	7.52	2.08	7.52

Panel B: estimates of cointegrating vectors

$\text{Ln}\, X_{u.s.}$	$\text{Ln}\, Y_{u.k.}$	$\text{Ln}\,(P_{u.s.} \cdot R/P_{u.k.})$	Constant
−1.00	5.48	3.49	−18.37
−1.00	2.07	−0.97	−6.74
[10.6]	[9.64]	[13.0]	[9.14]

Note: In panel B the degrees of freedom of χ^2 statistic is the same as number of cointegrating vectors, i.e., 2. The critical value of $\chi^2_{(2)} = 5.99$, at the 5% level of significance.

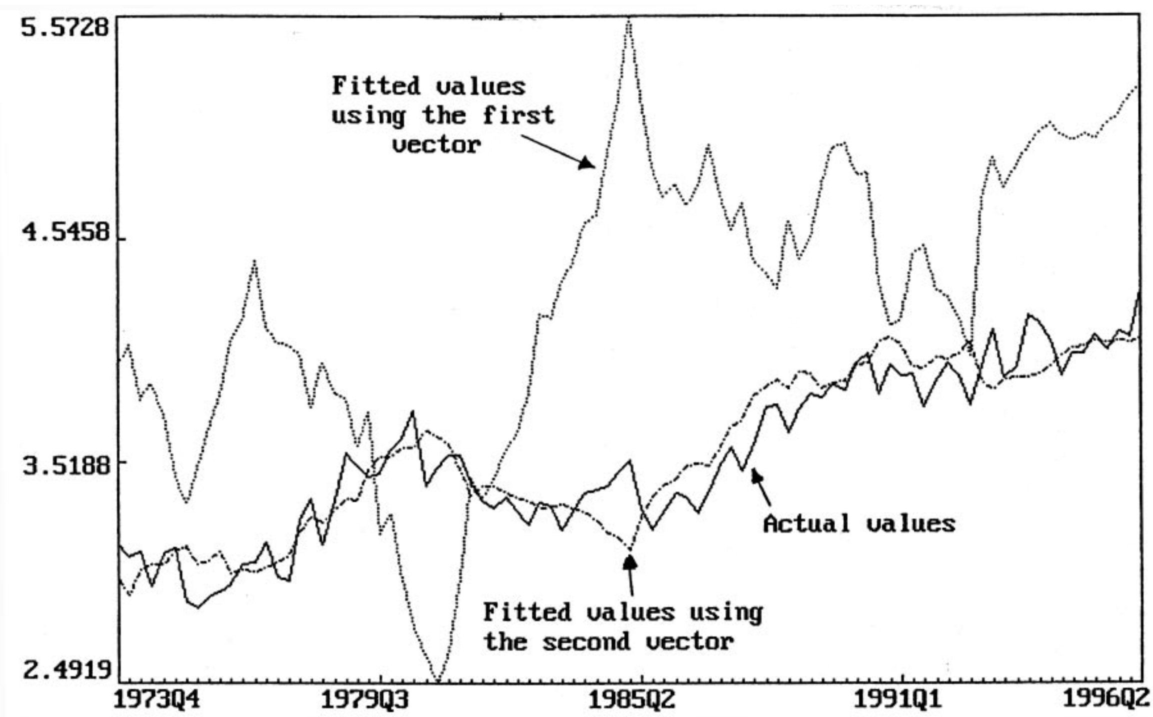

Fig. 2. *Fitted us actual values of export demand vectors when the order of VAR = 3*

ing to observe that the second vector is the vector that yields theoretically expected positive income elasticity and negative real exchange rate elasticity. From this second vector, it is obvious that UK demand for the US exports is income elastic (just like US import demand) and the relative price elasticity is very close to unity. From the second vector of Table 3 and the second vector of Table 4, it is gathered that the absolute value of the sum of import and export demand elasticities adds up to more than unity, indicating that real depreciation of the dollar against the pound will have a favourable effect on US trade with the UK.

For comparison purpose the AIC and SBC criteria are applied to select the lag length in this case too. Just like import demand case, the AIC selected two lags and the SBC, one lag.

IV. SUMMARY AND CONCLUSION

In applying the Johansen-Juselius cointegration technique there are two controversial issues. The first is the fact that the results are sensitive to the lag order. The second is related to the choice of a specific vector in case of evidence of multiple cointegrating vectors. In this paper the goodness of fit as a criterion not only for selecting the lag length in the Johansen-Juselius procedure is proposed, but also as a criterion for selecting one vector over the others in case of multiple vectors. In comparing this criteria to AIC and SBC, it was revealed that it selects a lag length that is different than those selected by AIC and SBC. If one is concerned with the predictive power of a model, then goodness of fit criteria should be preferred to others. Although the main purpose of the paper was to offer a criterion for the choice of lags and a specific vector, we thought to introduce the idea in the context of the US and the UK trade relations. Thus, unlike previous research, in the beginning disaggregated data was used to estimate bilateral trade elasticities. To this end, the existing trade models are first modified so that they can better fit to trade between two countries. The results that are based on the goodness of fit of each estimated cointegrating vector, basically support the notion that the real exchange rate between the USA and the UK is a significant factor in determining the bilateral trade.

REFERENCES

Bahmani-Oskooee, M. (1986) Determinants of international trade flows: the case of developing countries, *Journal of Development Economics*, **20**, 107–23.

Bahmani-Oskooee, M. (1995) The long-run determinants of the US trade balance revisited, *Journal of Post Keynesian Economics*, **17**, 457–65.

Bahmani-Oskooee, M. (1996) The black market exchange rate and demand for money in Iran, *Journal of Macroeconomics*, **18**, 171–6.

Bahmani-Oskooee, M. (1998) Cointegration approach to estimate the long-run trade elasticies in LDCs, *International Economic Journal*, **12**, 89–96.

Cheung, Y.-W. and Lai, K. S. (1993) Finite-sample sizes of Johansen's likelihood ratio tests for cointegration, *Oxford Bulletin of Economics and Statistics*, **55**, 313–28.

Engle, R. F. and Granger, C. W. J. (1987) Co-Integration and Error Correction: representation, estimation, and testing. *Econometrica*, 251–276.

Houthakker, H. S. and Magee, S. (1969) Income and price elasticities in world trade, *Review of Economics and Statistics*, **51**, 111–25.

Johansen, S. (1988) Statistical Analysis of Cointegration Vectors, *Journal of Economic Dynamics and Control*, **12**, 231–254.

Johansen, S. and Juselius, K. (1990) Maximum likelihood estimation and inference on cointegration-with application to the demand for money, *Oxford Bulletin of Economics and Statistics*, **52**, 169–210.

Khan, M. (1974) Import and export demand in developing countries, *IMF Staff Papers*, **21**, 678–93.

Kreinin, M. E. (1967) Price elasticities in international trade, *The Review of Economics and Statistics*, **49**, 510–16.

Kreinin, M. E. (1973) Disaggregated import demand functions – further results, *Southern Economic Journal*, **40**, 19–25.

Kwiatkowski, D., Phillips, P. C. B., Schmidt, P. and Shin, Y. (1992) Testing the null hypothesis of stationarity against the alternative of a unit root, *Journal of Econometrics*, **54**, 159–78.

Marquez, J. (1990) Bilateral trade elasticities, *The Review of Economics and Statistics*, **72**, 70–7.

Toda, H. Y. (1994) Finite sample properties of likelihood ratio tests for cointegrating ranks when linear trends are present, *Review of Economics and Statistics*, **76**, 66–79.

Warner, D. and Kreinin, M. E. (1983) Determinants of international trade flows, *Review of Economics and Statistics*, **65**, 96–104.

APPENDIX

Data definition and source

All data are quarterly over 1973I–1996II period and collected from the following sources:

1. International Financial Statistics of the International Monetary Fund, different issues.

2. Direction of Trade Statistics of the International Monetary Fund, different issues.

Variables:

$M_{u.s.}$ = The USA real imports from the UK. It is defined as the ratio of US imports in nominal dollars deflated by the US import price index. While the data on US nominal imports from the UK come from source 1 the US import price index (1990 = 100) comes from source 2.

$Y_{u.s.}$ = The US real GDP (in 1990 dollars). Data come from source 1.

$P_{u.s.}$ = The US GDP deflator (1990 = 100), from source 1.

$P_{u.k.}$ = The UK GDP deflator (1990 = 100), from source 1.

R = Nominal bilateral exchange rate defined as number of British pounds per US dollar. Data come from source 1.

$X_{u.s.}$ = The US real exports to the UK. It is defined as the ratio of US exports in nominal dollars deflated by the US export price index. While the data on US nominal exports to the UK come from source 2, the US export price index (1990 = 100) comes from source 1.

$Y_{u.k.}$ = The UK real GDP in 1990 pounds, source 1.